CSE 596 Lecture Mon 10/1 Fall 2018

Def^n. A language A is {recursively enumerable r.e. / computably enumerable <u>c.e.</u> / Turing-acceptable} if there is a DTM M such that $L(M) = A$.

If also M is <u>total</u>, ie. halts for all inputs ($\forall x\ M(x)\downarrow$) "halts", then A is {decidable / recursive / ~~recognizable~~}

A TM M <u>runs in time $t(n)$</u> if for all $x \in \Sigma^*$, {the computation / all computations} $M(x)$ halt <u>within</u> $t(|x|)$ steps. applies to NTMs

M runs in space $s(n)$ if $\forall x$, {the / all} computation(s) $M(x)$ uses space at most $s(|x|)$ (i.e., <u>changes</u> at most $s(|x|)$ different cells)

DTIME$[t(n)]$ = $\{A : A = L(M)$ for some DTM M that runs in time $t(n)$.$\}$

NTIME$[t(n)]$ = $\{A : A = L(N)$ for some NTM N ...

DSPACE$[s(n)]$ = $\{A : A = L(M)$ where M is a DTM that runs in space $s(n)$.

NSPACE$[s(n)]$ = $\{A : A = L(N)$ ... NTM N ... $\}$.

Generally we postulate that a <u>time function</u> $t(n)$ satisfies $t(n) \geq n+1$

Enough time to read an n-bit input x and allow 1 more step to halt

and a space function $s(n)$ is either constant or satisfies $s(n) \geq \frac{1}{c}\log_2 n$ for some $c > 0$.
ie. $s(n) = \underline{\Omega(\log n)}$

Important Cases:
Cases — Classes!;

$DLIN = DTIME[O(n)]$ deterministic linear time on a <u>Multi</u>-tape TM.

$NLIN = NTIME[O(n)]$ ~~Can be single tape (!) ignore why~~

$P = \bigcup_{k \geq 1} DTIME[O(n^k)]$ $= \{L(M) : M$ is a DTM that runs in some time $t(n)$ that is $\leq a \cdot n^k + a$ for some $a \geq$, $k \geq 1$, all n. $\}$

$NP = \bigcup_{k \geq 1} NTIME[O(n^k)]$ "<u>N</u>ondeterministic <u>P</u>olynomial Time"

$PSPACE = \bigcup_{k \geq 1} DSPACE[O(n^k)]$ "Polynomial Space"

$NPSPACE$ . . . . Next month: $= PSPACE$!

$\underline{L} = DSPACE[O(\log n)]$ det logspace $L \overset{?}{=} NL$ <u>unknown</u>

aka. DLOG or LOGSPACE

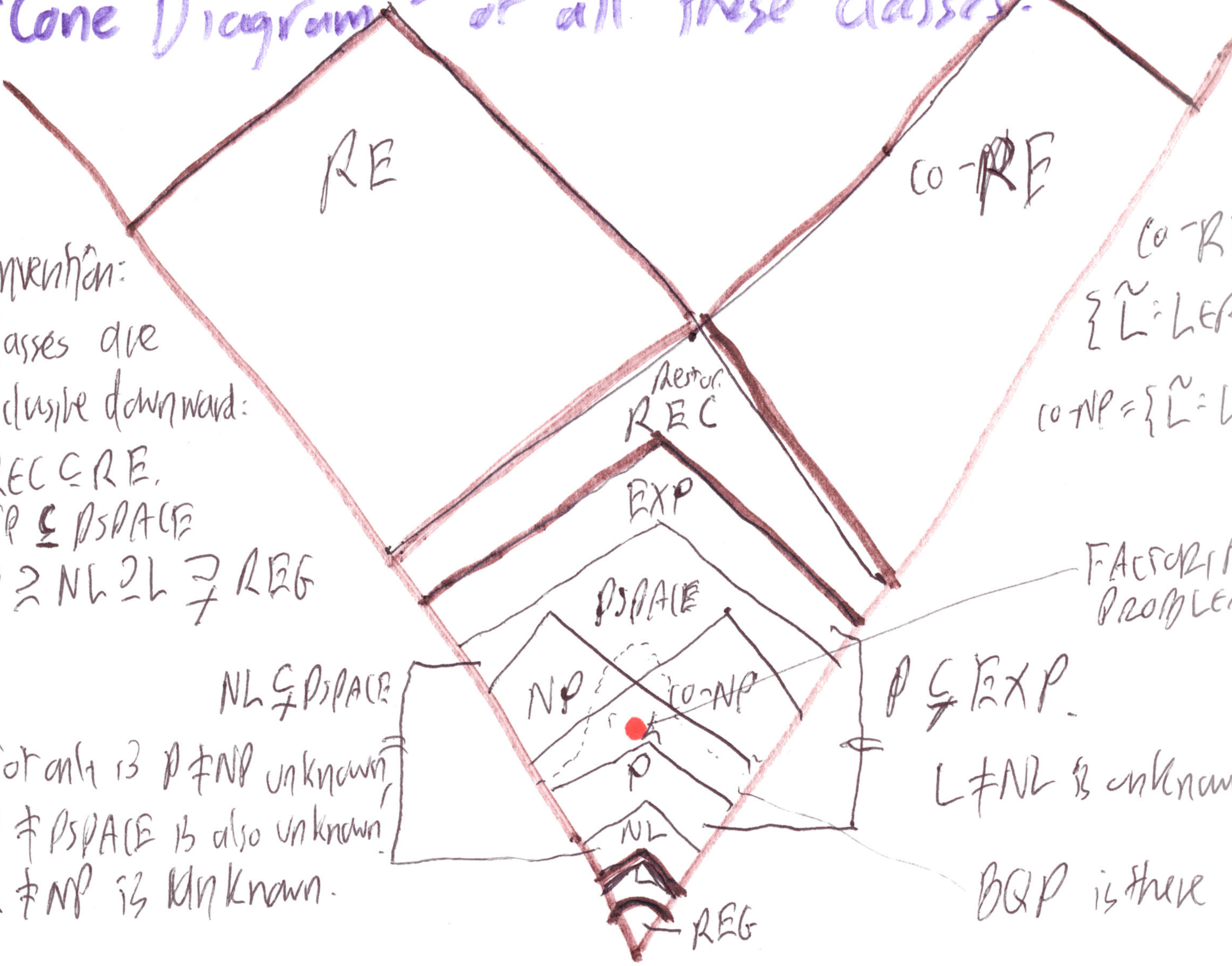
$NL = NSPACE[O(\log n)]$ Nondeterministic Logspace

$EXP = \bigcup_{k \geq 1} DTIME[2^{n^k}]$ "2 to the poly" Exponential Time

There is also <u>NEXP</u> and more.

$DSPACE[O(1)] \overset{?}{=} \underline{REG} \subsetneq DTIME(n+1) \subsetneq DLIN$

"Cone Diagram" of all these classes.



Convention: Classes are inclusive downward: $REC \subseteq RE$, $NP \subseteq PSPACE$, $P \supseteq NL \supseteq L \supsetneq REG$

$NL \subsetneq PSPACE$

Not only is $P \neq NP$ unknown, $P \neq PSPACE$ is also unknown. $L \neq NP$ is unknown.

$co\text{-}RE = \{\tilde{L} : L \in RE$

$co\text{-}NP = \{\tilde{L} : L \in NP$

$P \subsetneq EXP$.

$L \neq NL$ is unknown

Starting at the top end, the diagram communicates these relationships:

$REC \neq RE$, $co\text{-}RE \neq RE$, but $REC = RE \cap co\text{-}RE$.

Note for contrast: $P \subseteq NP$, $P \subseteq coNP$, but $NP \cap coNP$ is not known to $= P$.

REC is closed under complements. **ditto EXP, PSPACE, P, L, NL, REG, but not RE or co-RE, NP, co NP unknown.**

$co\text{-}REC = REC$, ie L decidable $\Rightarrow \tilde{L}$ decidable

Proof of this fact: Let any $A \in REC$ be given.

Then there is a DTM $M_A = (Q_A, \Sigma, \Gamma, \ldots, \{q_{acc}\}, q_{rej})$ s.t. $L(M_A) = A$ and for all x, $M(x)\downarrow$ — and halts either in q_{acc} or in q_{rej}. $\rightarrow \tilde{A} \in REC$. $\boxtimes$

Define $M' = (Q_A, \Sigma, \Gamma, \ldots, \{q_{rej}\}, q_{acc})$. Then $L(M') = \tilde{A}$ and M' is also total! So