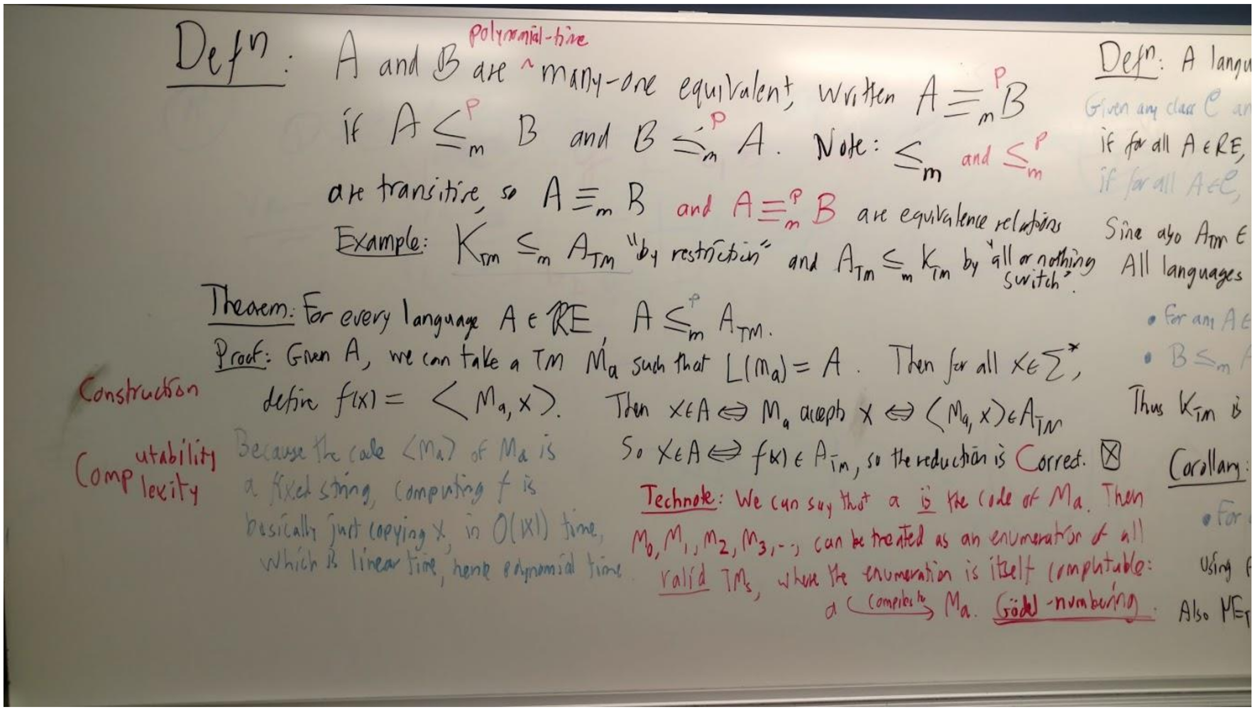
CSE596 Lecture Friday Oct. 4, 2019



Defn: A and B are polynomial-time many-one equivalent, written $A \equiv_m^p B$ if $A \leq_m^p B$ and $B \leq_m^p A$. Note: $\leq_m$ and $\leq_m^p$ are transitive, so $A \equiv_m B$ and $A \equiv_m^p B$ are equivalence relations.

Example: $K_{TM} \leq_m A_{TM}$ "by restriction" and $A_{TM} \leq_m K_{TM}$ by "all or nothing switch".

Theorem: For every language $A \in RE$, $A \leq_m^p A_{TM}$.

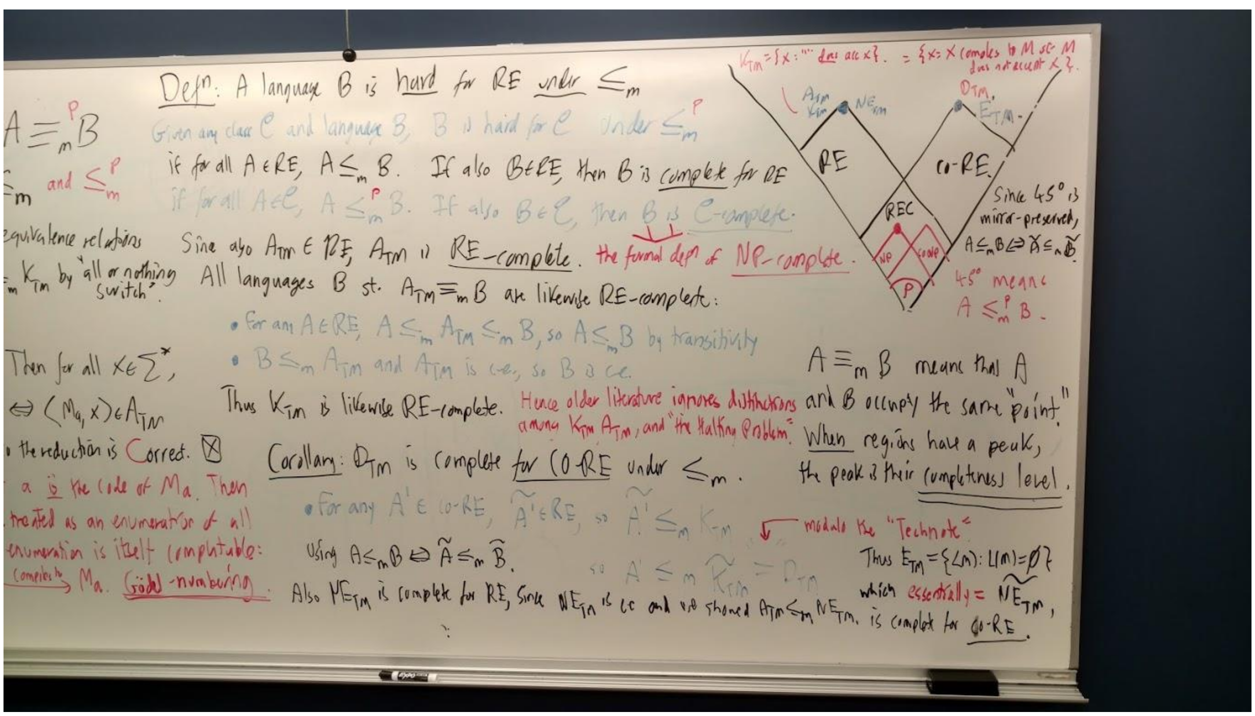
Proof: Given A, we can take a TM M_a such that $L(M_a) = A$. Then for all $x \in \Sigma^*$,

Construction: define $f(x) = \langle M_a, x \rangle$. Then $x \in A \iff M_a$ accepts $x \iff \langle M_a, x\rangle \in A_{TM}$

So $x \in A \iff f(x) \in A_{TM}$, so the reduction is Correct. ⊠

Computability / Complexity: Because the code $\langle M_a \rangle$ of M_a is a fixed string, computing f is basically just copying x, in $O(|x|)$ time, which is linear time, hence polynomial time.

Technote: We can say that a is the code of M_a. Then $M_0, M_1, M_2, M_3, \ldots$ can be treated as an enumeration of all valid TMs, where the enumeration is itself computable: $a \xrightarrow{\text{computable}} M_a$. Gödel-numbering.



$K_{TM} = \{x : \text{"x" does acc } x\}$. $= \{x : x \text{ compiles to } M \text{ s.t. } M \text{ does not accept } x\}$.

Defn: A language B is hard for RE under $\leq_m$ if for all $A \in RE$, $A \leq_m B$. If also $B \in RE$, then B is complete for RE.

Given any class $\mathcal{C}$ and language B, B is hard for $\mathcal{C}$ under $\leq_m^p$ if for all $A \in \mathcal{C}$, $A \leq_m^p B$. If also $B \in \mathcal{C}$, then B is $\mathcal{C}$-complete. the formal defn of NP-complete.

Since also $A_{TM} \in RE$, A_{TM} is RE-complete. All languages B s.t. $A_{TM} \equiv_m B$ are likewise RE-complete:

- For any $A \in RE$, $A \leq_m A_{TM} \leq_m B$, so $A \leq_m B$ by transitivity
- $B \leq_m A_{TM}$ and A_{TM} is c.e., so B is c.e.

Thus K_{TM} is likewise RE-complete. Hence older literature ignores distinctions among K_{TM}, A_{TM}, and "the Halting Problem".

Since 45° is mirror-preserved, $A \leq_m B \iff \tilde{A} \leq_m \tilde{B}$.

45° means $A \leq_m^p B$.

$A \equiv_m B$ means that A and B occupy the same "point". When regions have a peak, the peak is their completeness level.

Corollary: D_{TM} is complete for co-RE under $\leq_m$.

- For any $A' \in$ co-RE, $\tilde{A'} \in RE$, so $\tilde{A'} \leq_m K_{TM}$ modulo the "Technote"

Using $A \leq_m B \iff \tilde{A} \leq_m \tilde{B}$. so $A' \leq_m \tilde{K}_{TM} = D_{TM}$

Also NE_{TM} is complete for RE, since NE_{TM} is c.e. and we showed $A_{TM} \leq_m NE_{TM}$. Thus $E_{TM} = \{\langle M \rangle : L(M) = \emptyset\}$ which essentially $= \widetilde{NE_{TM}}$, is complete for co-RE.

Defⁿ: HP_{TM} is the problem INST: A TM M, and an input w to M. (default: M is deterministic)
QUES: Does $M(w)\downarrow$?

Clearly c.e.: we regard Turing Kit as accepting if it raises the "String not accepted" dialog too.

Hence to show $HP_{TM} \equiv_m A_{TM}$, it suffices to show $\boxed{A_{TM} \leq_m HP_{TM}}$.

$\langle M, w\rangle \xrightarrow{f} \langle M', w'\rangle$

$w' = w$ and $M' =$

input x

Simulate $M(w)$ if and when it emerges

if M accepts — halt

if M rejects — Ω? inf loop

For correctness, we need:

$\equiv$ by itself means "means"

Source is AP_{TM}. Target

$\langle M, w\rangle \in AP_{TM} \Longrightarrow$ $M(w)$ accepts $\Longrightarrow$ $M'(w)$ halts $\equiv \langle M', w\rangle \in HP_{TM}$.

$\langle M, w\rangle \notin AP_{TM} \Longrightarrow$ $\equiv$ $M(w)$ does not accept: it might reject or diverge $\uparrow$. $\Longrightarrow$ $M'(w)$ doesn't halt. $\equiv \langle M', w\rangle \notin HP_{TM}$

(if $M(w)$ rejects, $M'(w)$ loops at q_{rej} infinitely)

So $A_{TM} \leq_m HP_{TM}$, so HP_{TM} is likewise RE-complete.

The other choice works to show $HP_{TM} \leq_m A_{TM}$ directly instead — exercise.

How about $C = \{M : L(M) = \Sigma^*\}$?
The problem is called ALL_{TM}.

$A_{TM} \leq_m ALL_{TM}$ follows by the same reduction as for $A_{TM} \leq_m NE_{TM}$.
Monday will show $D_{TM} \leq_m ALL_{TM}$, which hence is neither c.e. nor co-c.e.

Defⁿ: A language B is hard for

Given any class $\mathcal{C}$ and language B, B is

if for all $A \in RE$, $A \leq_m B$. If als

if for all $A \in \mathcal{C}$, $A \leq_m^p B$. If al

Since also $A_{TM} \in RE$, A_{TM} is RE-

All languages B s.t. $A_{TM} \equiv_m B$

- For any $A \in RE$, $A \leq_m A_{TM} \leq$
- $B \leq_m A_{TM}$ and A_{TM} is c

Thus K_{TM} is likewise RE-complete.

Corollary: D_{TM} is complete f

- For any $A' \in$ co-RE,

Using $A \leq_m B \iff \tilde{A} \leq_m \tilde{B}$.

Also NE_{TM} is complete for RE, S