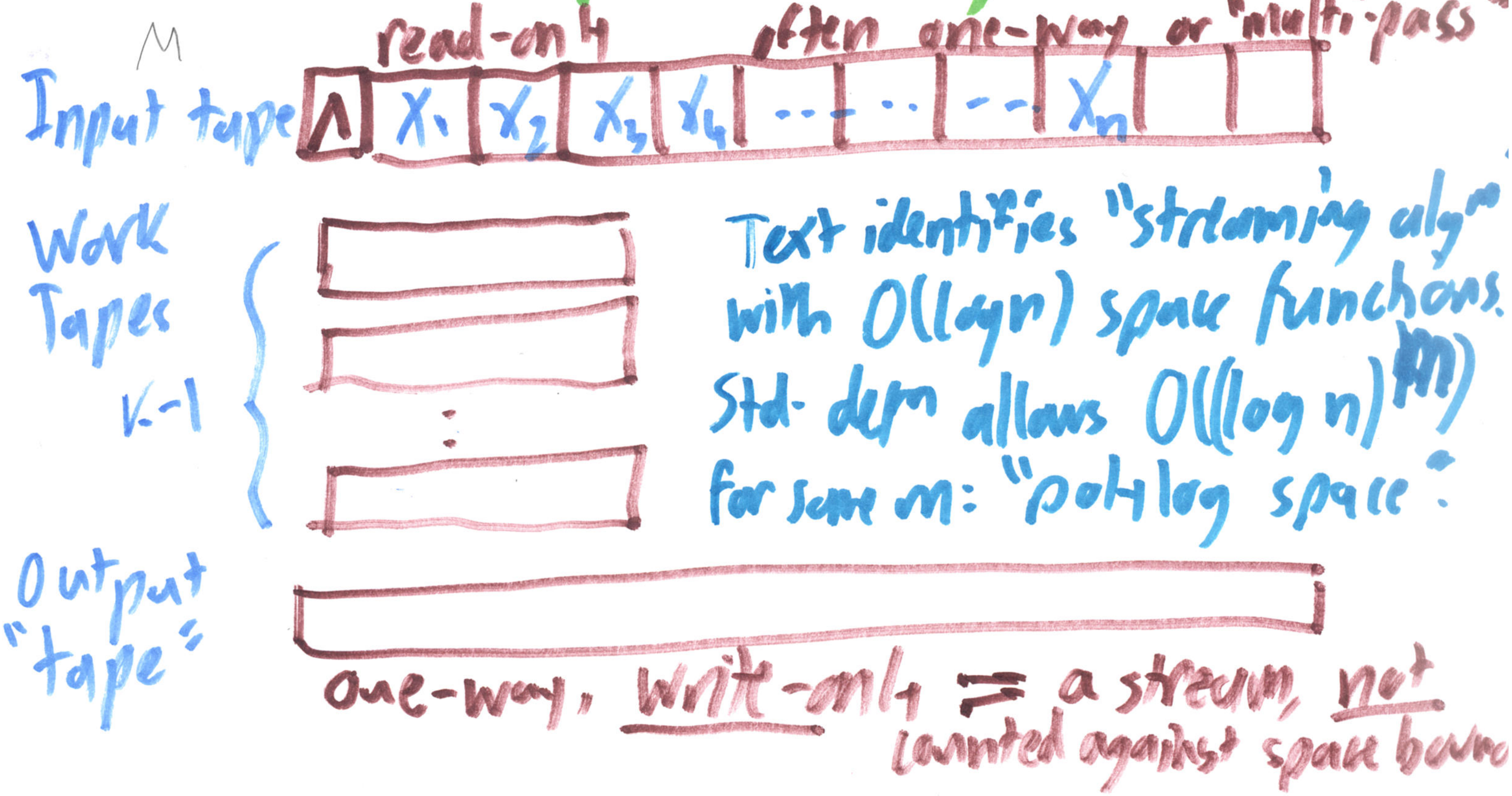
CSE596 Lecture Fri 10/5 Fall 2018

For any languages $A, B \subseteq \Sigma^*$, write $\underline{A \leq_m B}$ (m for "many-one" or "mapping") if there is a **computable** function $f: \Sigma^* \to \Sigma^*$ s.t.

for all x, $x \in A \iff f(x) \in B$. If f is polynomial-time computable we write $A \leq_m^p B$

If log-space computable, write $A \leq_m^{\log} B$ etc.

Note about the computation of functions.



Text identifies "streaming alg^m" with $O(\log n)$ space functions. Std. defn allows $O((\log n)^m)$ for some m: "polylog space".

Example: $K_{TM} = \{x : x \text{ codes } M \text{ s.t. } M \text{ accepts } x\}$

Then $A_{TM} = \{\langle M, x\rangle : M \text{ accepts } x\}$.

$K_{TM} \leq_m A_{TM}$ via the mapping $f(x) = \langle x, x\rangle$.

NE_{TM} : Instance: A DTM M.
Question: Is $L(M) \neq \emptyset$?

I.e. $(\exists x)$ M accepts x?

As a language:

$NE_{TM} = \{\langle M\rangle : L(M) \neq \emptyset\}$.

Theorem: NE_{TM} is c.e. but $A_{TM} \leq_m NE_{TM}$ so it is undecidable.

Proof: "Is c.e.": Imagine an NTM N that on input $\langle M\rangle$ guesses a string x, runs $M(x)$, and accepts if and when M accepts x. Then $L(N) = NE_{TM}$.

can convert N to DTM, so NE_{TM} is c.e.

Reduction: Map from A_{TM} instances

Map to codes of single machines $\langle M'\rangle$ which are instances of the NE_{TM} problem.

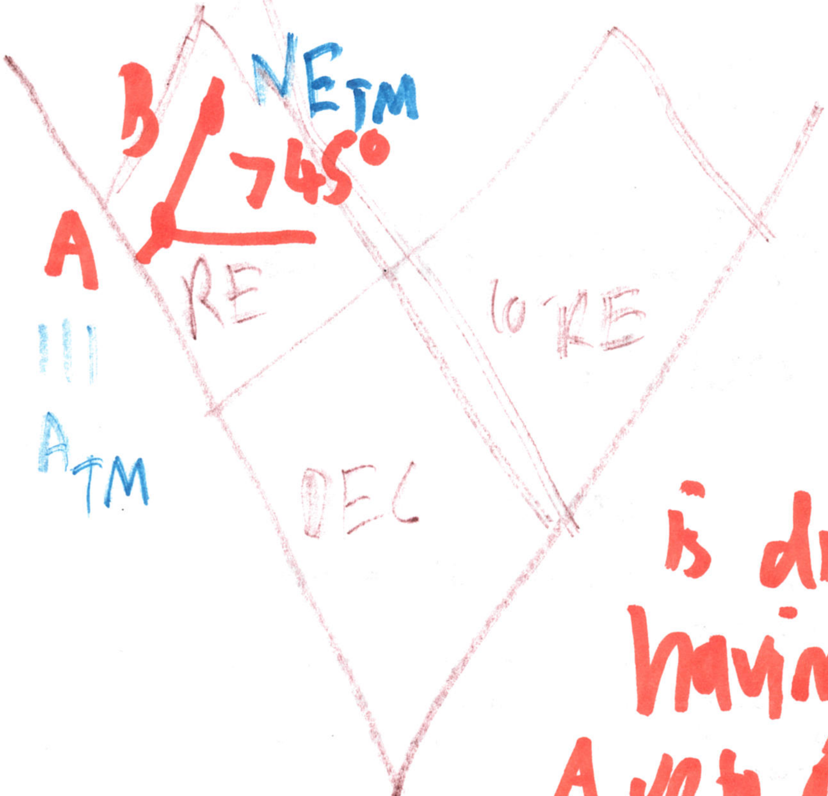
$\langle M, x\rangle \xrightarrow{f} M'$: ↓ input y

Simulate $M(x)$

if and when M accepts x

Accept y.

Then $\langle M, x\rangle \in A_{TM} \Rightarrow M$ accepts x $\therefore \langle M'\rangle \in NE_{TM}$
$\Rightarrow M'$ accepts every $y \Rightarrow L(M') = \Sigma^* \Rightarrow L(M') \neq \emptyset$
but $\langle M, x\rangle \notin A_{TM} \Rightarrow \forall y\ M'(y)$ never can accept
$\Rightarrow f\langle M, x\rangle = \langle M'\rangle \notin NE_{TM}$. So $A_{TM} \leq_m^{log} NE_{TM}$.



Convention: The relation

$$A \leq_m^{P \text{ or } \log} B$$

is diagramed by having the angle from A up to B be at least as steep as the cone walls

<u>Monday Preview</u>: This will follow from the "<u>Reduction Theorem</u>": Suppose $A \leq_m^{(?)} B$. Then:

- If B is decidable then A is decidable
- B is c.e. $\Rightarrow$ A is c.e.
- B is co-c.e $\Rightarrow$ A is co-c.e.

And when we know $A \leq_m^{P} B$:

- $B \in P \Rightarrow A \in P$
- $B \in NP \Rightarrow A \in NP$
- $B \in coNP \Rightarrow A \in coNP$.