

Define $ALL_{TM} = \{ \langle M \rangle : L(M) = \Sigma^* \}$. (Similar to $TOT_{TM} = \{ \langle M \rangle : \forall x M(x) \downarrow \}$) "Delay Switch"

We've seen $A_{TM} \leq_m ALL_{TM}$ v.a $\langle M, w \rangle \mapsto M'$

$\langle M, w \rangle \in A_{TM} \iff M \text{ accepts } w \implies \forall x M' \text{ accepts } x \iff \langle M' \rangle \in ALL_{TM}$

$\langle M, w \rangle \notin A_{TM} \implies \exists x M' \text{ does not accept } x \implies L(M') = \emptyset \implies \langle M' \rangle \notin ALL_{TM}$

Define the problem
 INST: A TM M
 Ques: Is $L(M)$ regular?
 The language is $\overline{I_{REG}}$.

Prac: $A_{TM} \leq_m \overline{I_{REG}}$. Pick $B = \{ 0^n 1^n : n \geq 1 \}$ which is non-regular.
 Edit the All-or-Nothing Switch to make it B or Nothing:
 $M \text{ accepts } w \implies \text{for all } x, M'(x) \text{ reaches the } B \text{ test} \implies L(M') = B \implies L(M') \text{ is not regular}$
 $M \text{ does not acc } w \implies L(M') = \emptyset \text{ as before, and } \emptyset \in REG$
 So $\implies L(M')$ is regular.

Construction For $\langle M \rangle \mapsto M'$
 $K_{TM} \leq_m ALL_{TM}$: Computable as a code mapping.
 "Alarm" (rej) "Normal Operations: no alarm" (acc)

Flowchart:
 input x
 Compute $n = |x|$
 Simulate $M(\langle M, w \rangle)$ for n steps only.
 Did $M(\langle M, w \rangle)$ accept in these n steps?
 yes: "Alarm" (rej) Reject x
 no: "Normal Operations: no alarm" (acc) Accept x

"Delay Switch"

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This construction also shows $D_{TM} \leq_m INF_{TM} = \{ \langle M \rangle : L(M) \text{ is infinite} \}$.
 We got $A_{TM} \leq_m INF_{TM}$ too.
 Hence all of ALL_{TM}, TOT_{TM} and INF_{TM} are neither ce- nor co-ce.

Abstract Def: For any class $C \subseteq RE$ define its index set to be $I_C = \{ \langle M \rangle : L(M) \in C \}$.

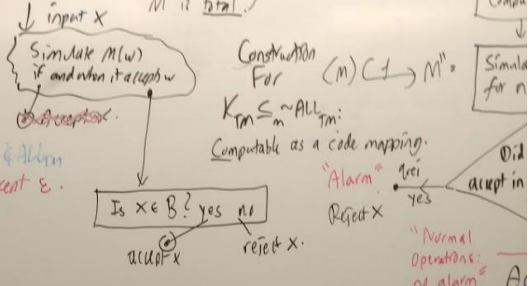
$ALL_{TM} = I_{\Sigma^*}$
 $INF_{TM} = I_{\{L : L \text{ is infinite}\}}$
 $E_{TM} = I_{\emptyset}$
 $I_{\emptyset} = \emptyset$
 $I_{REG} = \{ \langle M \rangle : L(M) \in REG \}$ = {all valid index} call it N_{TM}

Upward Cones for $A_{TM} \leq_m B$ and $D_{TM} \leq_m B$
 $\bullet ALL_{TM} \leq_m B$
 $\bullet INF_{TM} \leq_m B$

Diagram showing relationships between complexity classes: RE , $CO-RE$, REG , REC . Arrows indicate reductions like $A_{TM} \leq_m K_{TM}$ and $D_{TM} \leq_m B$.

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 $\langle M, w \rangle \in A_{TM} \iff M \text{ accepts } w \implies \forall x M' \text{ accepts } x \iff \langle M' \rangle \in ALL_{TM}$
 $\langle M, w \rangle \notin A_{TM} \implies \exists x M' \text{ does not accept } x \implies L(M') \neq \Sigma^* \implies \langle M' \rangle \notin ALL_{TM}$
 $\implies L(M') \text{ is finite}$
 $\implies M' \text{ accepts } \epsilon$
 $\implies M' \text{ does not accept } \epsilon$.



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$M \text{ accepts } w \implies \text{for all } x, M'(x) \text{ reaches the } B \text{ test}$
 $\implies L(M') = B \implies L(M') \text{ is not regular}$
 $M \text{ does not acc } w \implies L(M') = \emptyset \text{ as before, and } \emptyset \in REG$
 the B test is never reached. so $\implies L(M') \text{ is regular.}$