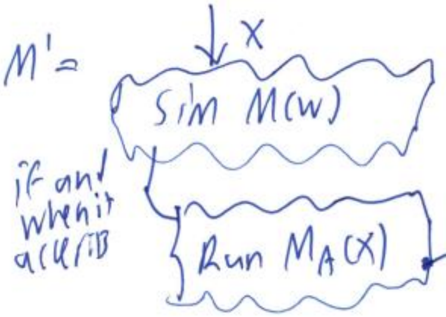
CSE596 Lecture Wed. Oct. 9

I forgot to snapshot the statement of Rice's Theorem and its proof before erasing the left board, but here is the same proof from last year's notes. The statement is that the only decidable index sets are those of the empty class (which gives the empty language) and of RE (which is Sigma^*).

<u>Proof</u>: Given $I_\mathcal{C}$ where $\mathcal{C} \neq \emptyset$ and $\mathcal{C} \neq RE$, so there is a c.e. language A such that either: (a) $\emptyset \in \mathcal{C}$, $A \notin \mathcal{C}$ (*)

Take an M_A s.t. $L(M_A) = A$. or (b) $A \in \mathcal{C}$, $\emptyset \notin \mathcal{C}$.

Build $\langle M, w \rangle \xrightarrow{f} M' =$



In case (a),

$\langle M,w\rangle \in A_{TM} \Rightarrow L(M') = A \Rightarrow M' \notin I_\mathcal{C}$ $\therefore A_{TM} \leq_m \tilde{I}_\mathcal{C}$.

$\langle M,w\rangle \notin A_{TM} \Rightarrow L(M') = \emptyset \Rightarrow M' \in I_\mathcal{C}$

In case (b)

$\langle M,w\rangle \in A_{TM} \Rightarrow L(M') = A \Rightarrow L(M') \in \mathcal{C} \Rightarrow M' \in I_\mathcal{C}$ $\therefore A_{TM} \leq_m I_\mathcal{C}$

$\notin A_{TM} \Rightarrow L(M') = \emptyset \Rightarrow M' \notin I_\mathcal{C}$ since $\emptyset \notin \mathcal{C}$ in this case

Either way, $I_\mathcal{C}$ is undecidable. ⊠

Example: $\mathcal{C} = REG$, case (b) applies with $A = \{\text{palindromes}\}$, so I_{REG} is undecidable.

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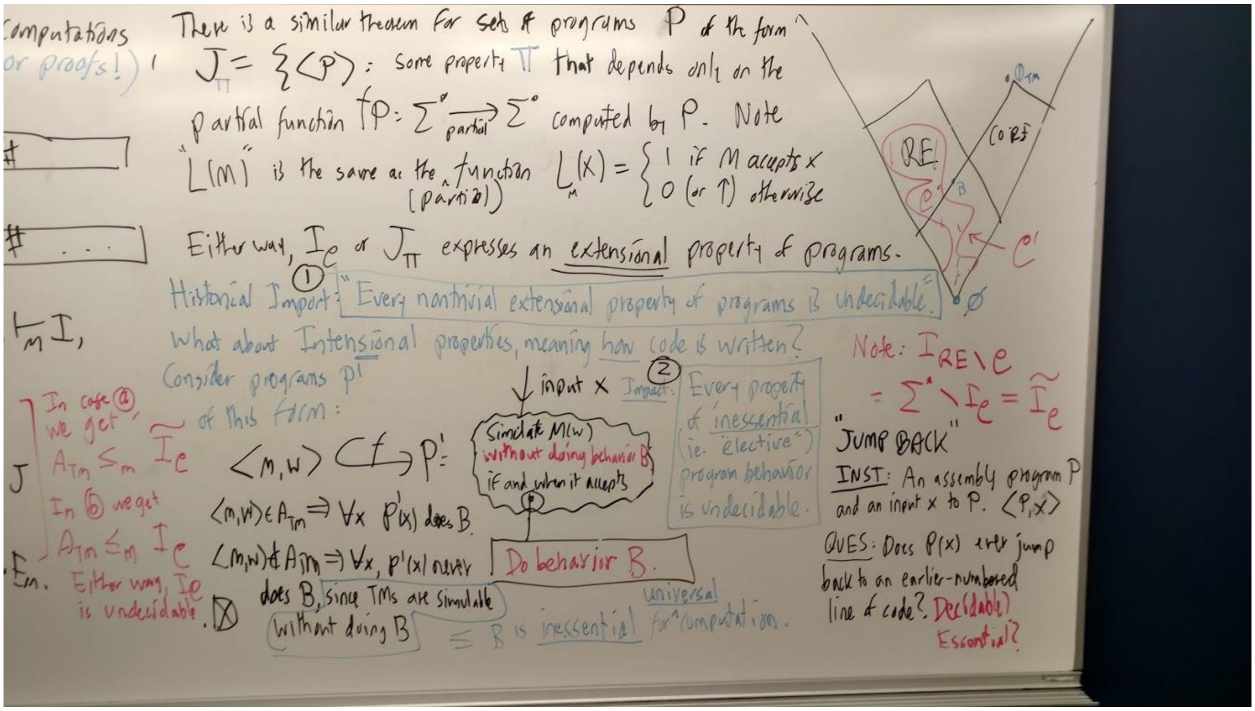
You can see the conclusions about the reductions in the right-hand board snap as the lecture continued:



Ignore that part in red pen in the last snap: