CSE 596 — Lecture Mon 10/15 — Fall 2018

Three Types of Reductions:
1. "Wait for G"
2. "All or Nothing Switch"
3. "Delay Switch"

COFFEE: Instance: A TM or Java program P with alphabet ASCII.

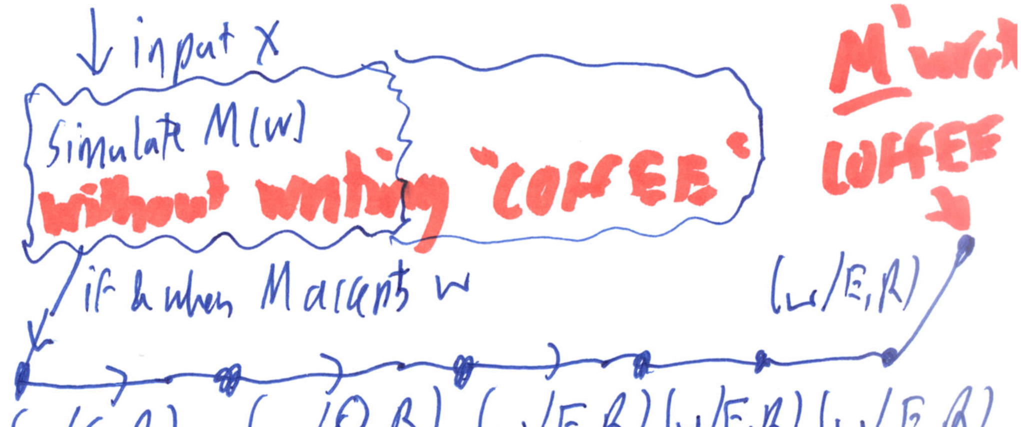
Question: Is there an input $x \in \Sigma^*$ such that $P(x)$ eventually writes "COFFEE" (on a tape or to std. output)?

Theorem: The language $L_{COFFEE} = \{\langle P\rangle : (\exists x) : P(x)$ outputs "COFFEE"$\}$ is c.e. but not decidable.

Proof: "Is c.e.": take an NTM N that given $\langle P\rangle$ as input guesses x, runs $P(x)$, and accepts if & when $P(x)$ = "COFFEE". We can convert the NTM N into a DTM M s.t. $L(M) = L(N) = L_{COFFEE}$.

Undecidable: Show that $A_{TM} \leq_m L_{COFFEE}$ via the following reduction f:

$\langle M, w\rangle \xrightarrow{f} M'$ (or call it P):



This f is computable by simple code edits adding these states.

For Java Pgm P, just add a line `System.out.println("COFFEE")` where the "Turing Kit" shows the "String Accepted" box.

Correctness: For all $\langle M, w\rangle$:

$\langle M,w\rangle \in A_{TM} \implies M$ accepts $w \implies$ for some x, $M'(x)$ writes "COFFEE" $\implies M' \in L_{COFFEE}$ (↗ for all inputs x, $M'(x)$ writes "COFFEE")

$\langle M,w\rangle \notin A_{TM} \equiv M$ never accepts $w \implies$ for all x, M' never writes "COFFEE" $\implies M' \notin L_{COFFEE}$

So $A_{TM} \leq_m L_{COFFEE}$ via f, so L_{COFFEE} is undecidable. ☒

General "Moral": Any **inessential** program behavior is undecidable.

You can write a program like Turing Kit that never exhibits that feature or behavior.

Example Behaviors:

① Turing Machine M' makes 3 Right moves in a row.

② M' never moves L? Essential, otherwise no power beyond DFAs.

decidable! (on a given X, that is.)

③ P is an assembly program with numbered instructions, and the behavior is P(x) JMPing to a lower-numbered instruction.

decidable: Just run P(x). Either it jumps to a lower instruction, whereupon you say "yes", or you exit over the last instruction, so answer is no. Because this is decidable, "JMP back" is essential!

INST: P, X

QUES: Does P(x) ever jump back?

Decidable "primitively"

INSTANCE P (just a program)

QUES: Does there exist X st P(x) jumps back?

Is it decidable?? Study Q.

2. All or Nothing Switch: $A_{TM} \leq_m ALL_{TM}$.

$\langle M, w \rangle \xrightarrow{f} M'$

↓ input x

Ignore x for now. Simulate $M(w)$ if & when it accepts

Do something with x, such as Accept x.

Can do $\langle M \rangle \xrightarrow{g} M'$: Simulate $M(\langle M \rangle)$ get $K_{TM} \leq_m ALL_{TM}$.

since $L(M') \neq \emptyset$

$M' \in NE_{TM}$

⇑

$M' = f(M, w) \in ALL_{TM}$

$\langle M, w \rangle \in A_{TM} \equiv M$ accepts $w \Rightarrow$ for all x, $M'(x)$ accepts $\Rightarrow \underline{L(M') = \Sigma^*}$

$\langle M, w \rangle \notin A_{TM} \equiv M$ does not acc $w \Rightarrow$ for all x, M' never accepts x.

$\Rightarrow L(M') = \emptyset \Rightarrow M' \notin ALL_{TM}$.

So $A_{TM} \leq_m NE_{TM}$ as before, too.

$\Rightarrow \langle M' \rangle \notin NE_{TM}$

Flipping around to complements this also shows $D_{TM} \leq_m E_{TM}$

E_{TM}:

INST: A TM M

QUES: Is $L(M) = \emptyset$?

Complementary question to NE_{TM}.

Other things to do with x: e.g. accept x iff x is a palindrome

↪ other reductions and Rice's Theorem.

Wed: that and the Delay Switch. Preview:

$\langle M, w \rangle \xrightarrow{f''} M''$ s.t.

$\langle M, w \rangle \in A_{TM} \Rightarrow$ for all x, M'' accepts x iff x is a palindrome $\Rightarrow L(M'') = PAL \Rightarrow L(M'')$ is not regular

$\langle M, w \rangle \notin A_{TM} \Rightarrow$ for all x, M'' never accepts $x \Rightarrow L(M'') = \emptyset \Rightarrow L(M'')$ is regular.

So the problem $REGULAR_{TM}$, "Given a TM M, is $L(M)$ regular?" is undecidable