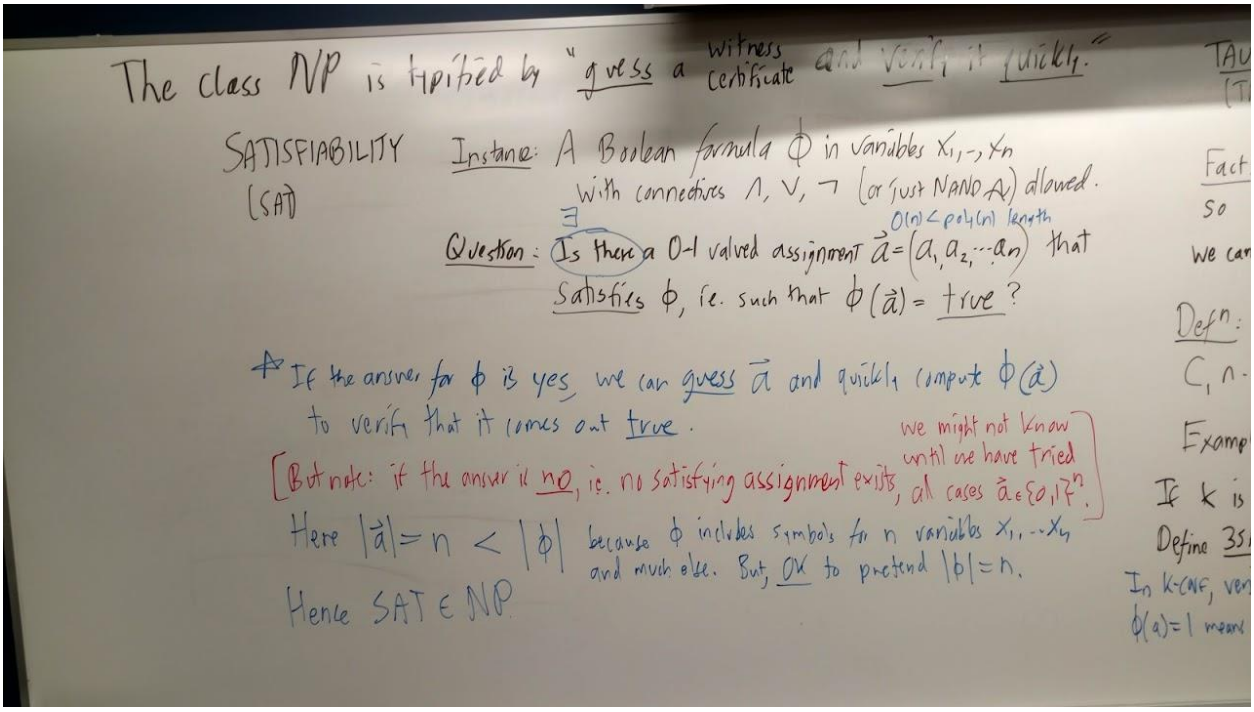
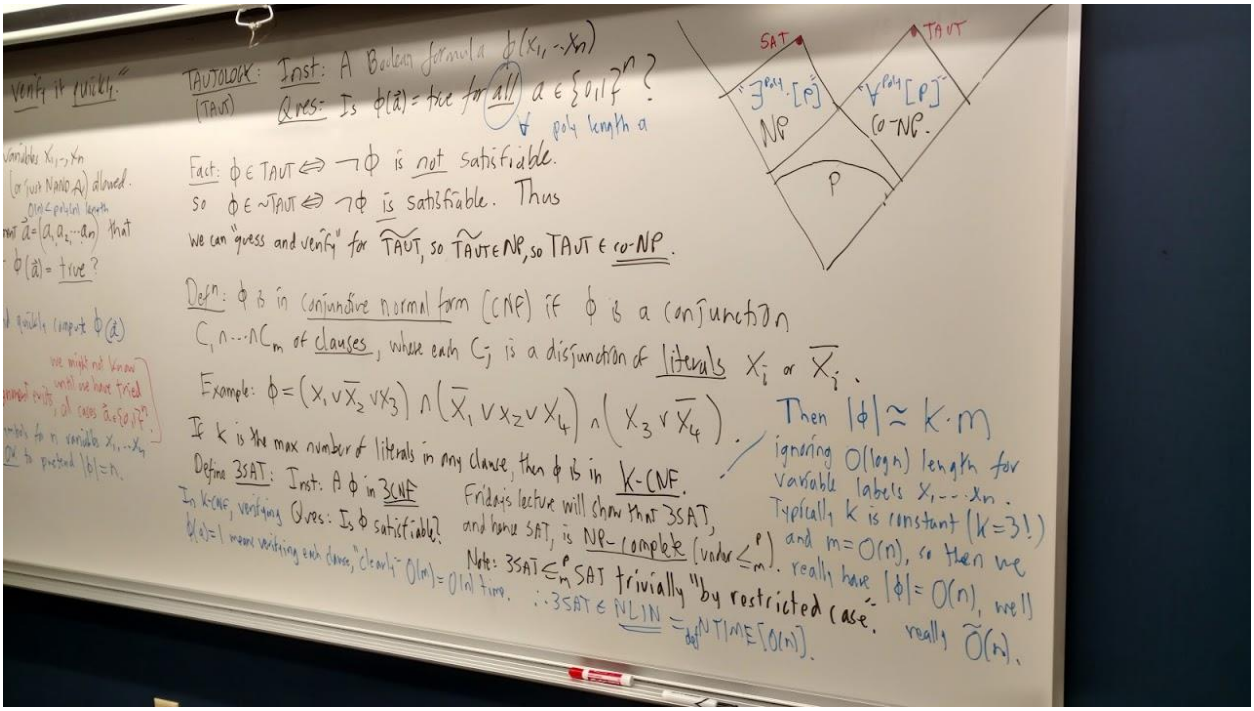
CSE596 Lecture Wed. Oct. 16, 2019



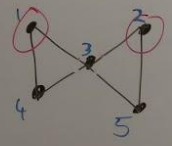


Other Problems in NP: Big $\exists$ quantifier over 2^n possible subsets ... small quantifier over at most n^2 possibilities

INDEPENDENT SET: Inst: An undirected graph $G=(V,E)$, an integer $K \geq 1$.

Question: Does there exist a subset $S \subseteq V$, $|S| \geq K$, such that no two nodes in S are connected by an edge, i.e. $\bigwedge_{u,v \in S} \neg E(u,v)$?

Poly length witness and can we find

eg. $G =$ 

Answer for $\langle G, 1 \rangle$ (M, w) is yes $S = \{3\}$ (silly but it works)

Answer for $\langle G, 2 \rangle$ is yes $S = \{1,2\}$ (or three other possibilities)

Answer for $\langle G, 3 \rangle$ is no.

Anyway, $|S| \leq n =_{\text{def}} |V|$ but we sometimes pretend n is the size of G.

If G has m edges, $|G|$ is better defined as $\sim m$, but often G is sparse meaning $m = O(n)$ anyway.

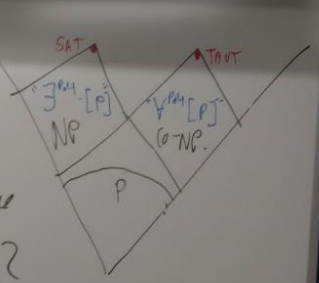
Verifying that S is independent takes time $\simeq O(|S|^2)$ going through each possible pair u,v from S.

Form is Big $\exists$ poly length as [poly time loop over u,v] $\therefore$ so $\in NP$.

TAUTOLOGY (TAUT): Inst: A Boolean formula $\phi(x_1, \ldots x_n)$

Ques: Is $\phi(\vec{a}) =$ true for all $a \in \{0,1\}^n$?

$\forall$, poly length a



CLIQUE: Inst $\langle G, k \rangle$ Same type of instance as for IND SET.

Ques: Is there $S \subseteq V$, $|S| \geq k$, such that $\bigwedge_{u,v \in S} (u,v) \in E$?

(then S is called a clique in G.)

$\therefore$ CLIQUE is in NP. $\exists$ so $|S| \leq |V|$ poly time

In fact, CLIQUE is equivalent to IND SET on mapping $G \mapsto G'$ = the complementary graph.

Study: Why different from TAUT being in co-NP?

HAM CIRCUIT: Inst: Just a graph $G=(V,E)$

Question: Is there an ordering $i_1, \ldots i_n$ of $\{1, \ldots n\}$ s.t. for all j, $(i_j, i_{j+1}) \in E$ and also $(i_n, i_1) \in E$ to complete the circuit? Also in NP.