

CSE596 Lecture Friday 10/18---the first word in blue is supposed to be "Computational":

Computational Complexity Theory is the study of how much time and space ^{ie memory} and other resources ^{is required to} solve computational problems. For individual ^{relations between pairs of} problems it has been ^{an unexpected failure} a resounding success.

Defn: For any time function $t(n) \geq n+1$ and space function $s(n) \geq \lceil \log_2 n \rceil$ define:

$DTIME[t(n)] = \{ L(n) : M \text{ is a DTM s.t. for all } x, M(x) \text{ halts within } t(|x|) \text{ steps} \}$

$NTIME[t(n)] = \{ L(n) : N \text{ is an NTM s.t. for all } x \text{ and all computation branches } \tilde{c}, \|\tilde{c}\| \leq t(|x|) \}$

$DSPACE[s(n)] = \{ L(n) : M \text{ is a DTM and for all } x, M(x) \text{ uses at most } s(|x|) \text{ cells that are read-writable} \}$

$NSPACE[s(n)] = \{ L(n) : N \text{ is an NTM and } (\forall x) \text{ for each branch } \tilde{c}, \tilde{c} \text{ uses at most } s(|x|) \text{ read-writable cells} \}$

It is OK to put O -notation inside the square brackets, e.g. $DTIME[O(n)]$ or $DTIME[n^{O(1)}]$.

Generally, for any class \mathcal{F} of time (or space) functions, $DTIME[\mathcal{F}]$ means $\bigcup_{f \in \mathcal{F}} DTIME[f(n)]$.

There are theorems allowing us to remove an outermost $O(\dots)$ but they can be misleading so ignore them.

$DTISP[t(n), s(n)] = \{ L(n) : M \text{ is a DTM and } \forall x, M(x) \text{ halts within } t(n) \text{ steps using no more than } s(n) \text{ RW cells} \}$

Handwritten notes on the right side of the board:
 (u,w) (v,w)
 If the input tape is read-only, this counts the worktape cells used.
 writing presumes \tilde{c} halts the entire computation.
 number of IDs that are read-writable.
 read-writable cells.
 Immediate Theorem: $DTISP[t(n), s(n)] \subseteq DTIME[t(n)] \cap DSPACE[s(n)]$
 Whether equality (over) holds is a subtle question!
 Consider TRIANGLE \equiv 3-CLIQUE.
 In: A simple graph G.
 Ques: Does G have a triangle?
 In P, for $q=1$ to n :
 In fact, for $w=1$ to n :
 if $(u,v) \in E \wedge (v,w) \in E \wedge (w,u) \in E$:
 Accept.

The Most Basic Complexity Classes:

$P = DTIME[n^{O(1)}] = UTIME[n^k, k]$

$NP = NTIME[n^{O(1)}] = \bigcup_{k \geq 1} NTIME[n^k, k]$

$PSPACE = DSPACE[n^{O(1)}]$

$NPSPACE = NSPACE[n^{O(1)}]$

Theorem = PSPACE! (proof later)

$LSPACE = DSPACE[O(\log n)]$ also called L

$NL = NSPACE[O(\log n)]$

Nobody writes $NLSPACE = LSPACE$? Not known!

$EXP = DTIME[2^{n^{O(1)}}]$

$NEXP = NTIME[2^{n^{O(1)}}]$

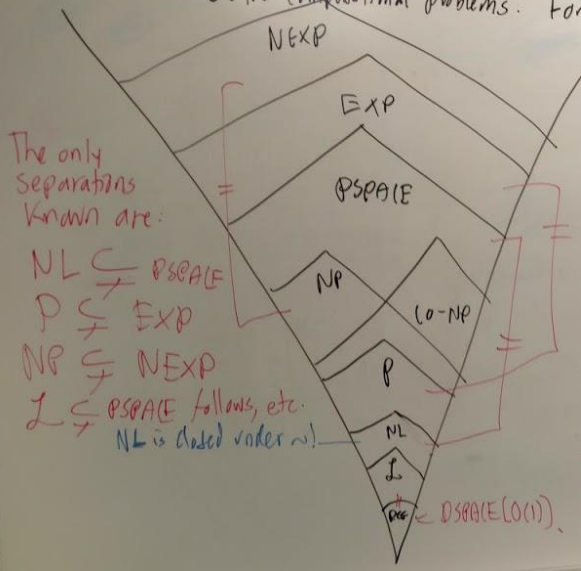
Runtime = $O(n^3) = o(n^{1.61})$

Diagram: A transducer, ie. a TM that computes a function, is allowed to have a special output tape that is write-only and so does not count against the space bound. Thus these defns work for classes of functions too. Sometimes we subscript "P" as in $DTIME_P[n^{O(1)}]$ but often not.

Diagram: A tape with input x_1, \dots, x_n and worktapes u, v, w, y . $m = \#x_i$ or $\#$ of edges in a vertex v_i . Indices of cells, counts of chars and node labels use all $O(\log n)$ space.

Handwritten notes on the left side of the board:
 and other resources is required to
 an unexpected failure.
 a resounding success.
 $s(n) \geq \lceil \log_2 n \rceil$ define:
 If the input tape is read-only, this counts the worktape cells used.
 $t(x)$ steps $\tilde{c}, \|\tilde{c}\| \leq t(|x|)$
 number of IDs
 (1) cells that are read-writable \tilde{c} .
 it must $s(|x|)$ read-writable cells.
 $DTIME[n^{O(1)}]$
 $DTIME(f(n))$
 ignore them
 using no more than $s(n)$ RW cells

Computational Complexity Theory is the study of how much time and space ^{ie memory} ^{and other resources} are required to solve computational problems. For individual relations between pairs of problems it has been an unexpected failure of a resounding success.



The only separations known are:
 $NL \subsetneq PSPACE$
 $P \subsetneq EXP$
 $NP \subsetneq NEXP$
 $L \subsetneq PSPACE$ follows, etc.
 NL is listed under L.

We can define det^C logspace-computable functions, hence logspace reductions $\leq \log_m$, but poly-time reductions $\leq P_m$ are mapped correctly at P and above for the $4S^0 \cdot B$ means $A \leq_m^P B$ rule.

For Wednesday please read §3 of ALR ch 27 on Boolean circuits, and §§ 1-3 of ALR ch 28.

There are classes $DSPACE(S(n))$ with $S(n)$ between $\log(\log n)$ and $\log n$, but ignore.

(U,V)
 (V,W)
 writing pressure
 E holds the entire
 Immediate T
 Whether eq
 Consider T
 Incl. A simple
 Ques. Dues