CSE596 Lecture Fri 10/19 Fall 2018

Example

- $L_\varepsilon = \{\langle M \rangle : M \text{ accepts } \varepsilon\} = \{\langle M \rangle : \langle M, \varepsilon \rangle \in A_{TM}\}$

  $= I_{\{L : \varepsilon \in L\}}$.
  - Hence undecidable.
  - Is c.e.: Just run M_U on $\langle M, \varepsilon \rangle$.

- $I_{\{\{\varepsilon\}\}} = \{\langle M \rangle : L(M) = \{\varepsilon\}\}$

  i.e. s.t. M accepts ε but does not accept any other string.

  Index set of the class containing only $\{\varepsilon\}$.

$A_{TM} \leq_m I_{\{\{\varepsilon\}\}}$ $\therefore$ undecidable, but let's see the reduction

$\langle M, w \rangle \mapsto M' =$ $\downarrow$ input x
{ Sim $M(w)$.
if & when accepts
$\downarrow$
if $x == \varepsilon$ accept else reject.

$\langle M, w \rangle \in A_{TM} \Rightarrow M'$ accepts and nothing else $\Rightarrow M' \in I_{\{\{\varepsilon\}\}}$

$\langle M, w \rangle \notin A_{TM} \Rightarrow L(M') = \emptyset$
$\Rightarrow L(M') \neq \{\varepsilon\} \Rightarrow M' \notin I_{\{\{\varepsilon\}\}}$

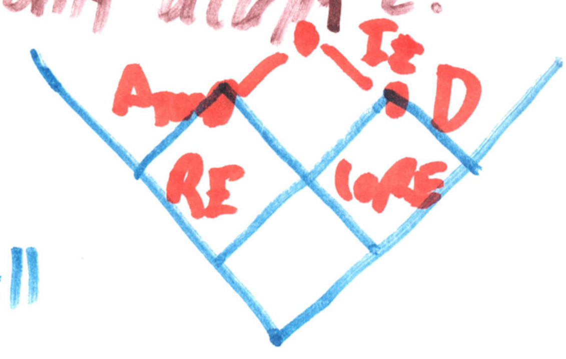
(Does also show $A_{TM} \leq_m L_\varepsilon$)

M accepts $w \Rightarrow L(M') = \{\varepsilon\}$ i.e. M' only accepts ε.

otherwise $\Rightarrow$ M' doesn't only accept ε.

- $I_{\{\{\varepsilon\}\}}$ is not c.e.:

  show $D_{TM} \leq_m I_{\{\{\varepsilon\}\}}$ as well



Delay Trick

$M \xrightarrow{g} M''$ =

↓ input x, $n = |x|$

[if $x == \varepsilon$, accept]

[Run $M(\langle M \rangle)$ for up to n steps]

↓ if it accepted, accept x.

M does not acc $\langle M \rangle \Rightarrow L(M'') = \{\varepsilon\}$

M does acc $\langle M \rangle \Rightarrow L(M'') \neq \{\varepsilon\} \Rightarrow M''$ does not only accept ε.

Does this give $D_{TM} \leq I_{\{\{\varepsilon\}\}}$ if we use a fuzzy box?

$M'' =$

↓ input x

[if $x = \varepsilon$ accept]

↓ else

[Run $M(\langle M \rangle)$] — if & when it accepts, accept x.

(so yes)

Checking Computations: An ID (instantaneous description) of a 1 tape TM M during a computation on input x is

$$I = \langle q, w, i \rangle$$

state, tape contents, cell #.

One can also write $I = uqcv$ where $w = ucv$ and M is scanning c.

or $I = u\underbrace{\begin{bmatrix} q \\ c \end{bmatrix}}_{\text{cell } i}v$.

$T(M, x, \vec{c})$: $\vec{c}$ is a sequence of IDs $I_0, I_1, \ldots, I_t$ s.t. $I_0 = \langle s, x, 1 \rangle$ (start ID on input x) and for all $j \geq 1$, I_j follows from I_{j-1} via the code of M.

Decidable in linear time!