CSE596 — Lecture Fri. Nov 2 — Fall 2018

GRAPH 3-COLORING (G3C) INST: Just a (undirected) graph $G = (V, E)$

QUES: Can G be 3-colored, i.e., is there a 3 function $\chi : V \to \{R, G, B\}$ s.t. $\forall (u,v) \in E,\ \chi(u) \neq \chi(v)$.

chi for chromatic

Verify in $\tilde{O}(m)$ time, $m \leq \binom{n}{2}$ $n=|$

$\therefore$ G3C $\in$ NP.

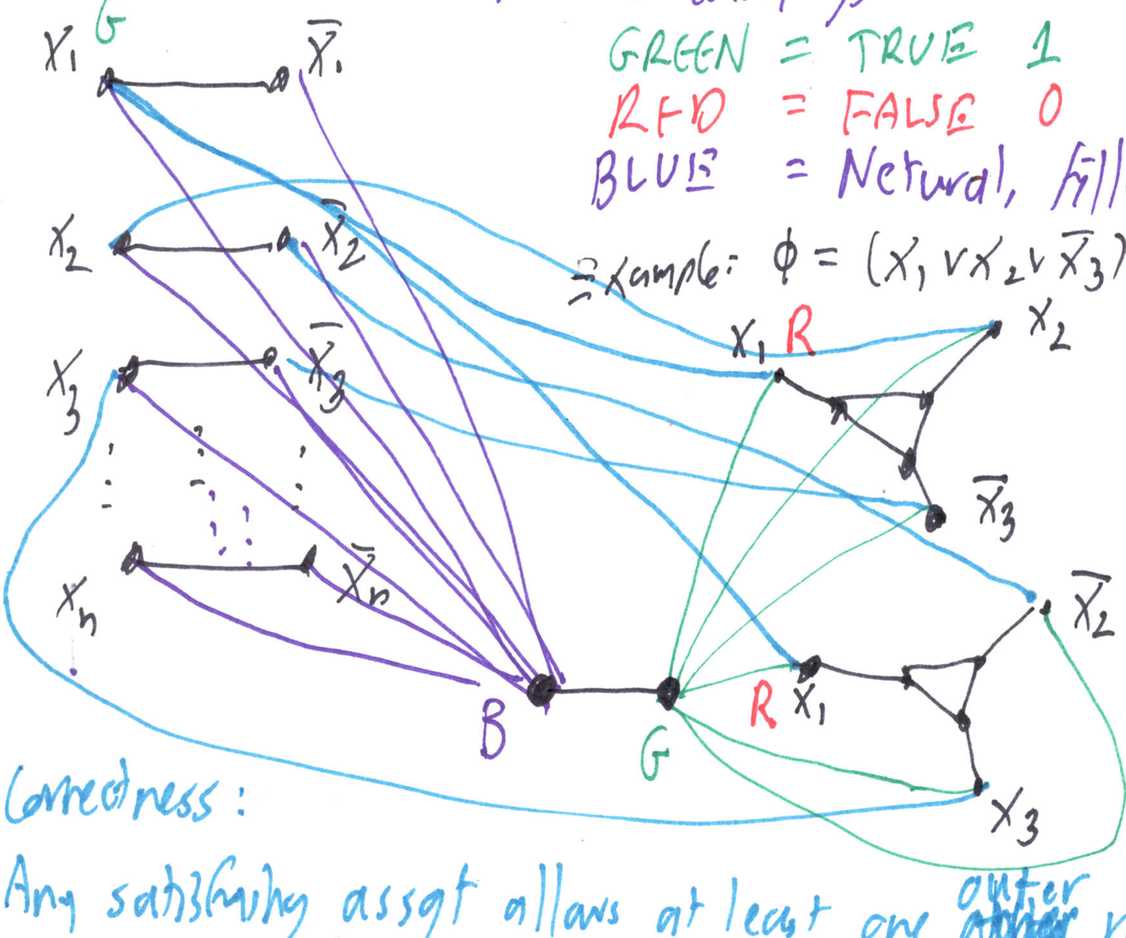
$3SAT \leq_m^p G3C$ First idea: establish a correspondence between possible colorings and truth assignments.

GREEN = TRUE 1

RED = FALSE 0

BLUE = Netural, filler, not critical value

Example: $\phi = (x_1 \vee x_2 \vee \bar{x}_3) \wedge (x_1 \vee \bar{x}_2 \vee x_3)$



Subject to this interpretation, the crossing edges from X_i in C_j (node x_{ij}) go to rung X_i with the same sign, likewise $\bar{X}_{ij}$ to $\bar{X}_i$.

Correctness:

Any satisfying assgt allows at least one ~~other~~ outer node in each clause gadget to be colored Red, which allows G_ϕ to be 3-colored. Vice-versa if G_ϕ has a 3-coloring then some outer node in each clause gadget must be red, and making the compatible rung node Green thus gives an assgt that satisfies all clauses, so $G_\phi \in G3C \iff \phi \in SAT$. $\boxtimes$

DIST CONNECTING PATHS

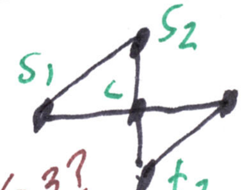
instance = $\langle G; s_1,\dots,s_K; t_1,\dots,t_K\rangle$ object.

INST: A undirected graph G with some number K of nodes $s_1,\dots,s_K$ labeled "sources" and K other nodes $t_1,\dots,t_K$ as their "targets".

QUES: Can we draw K paths from each s_i to its t_i so that no two paths meet at any vertex?

Case $K=1$: solve by BFS since no conflict is possible. $\therefore$ in P.

Case $K=2$: Impossible for $G =$



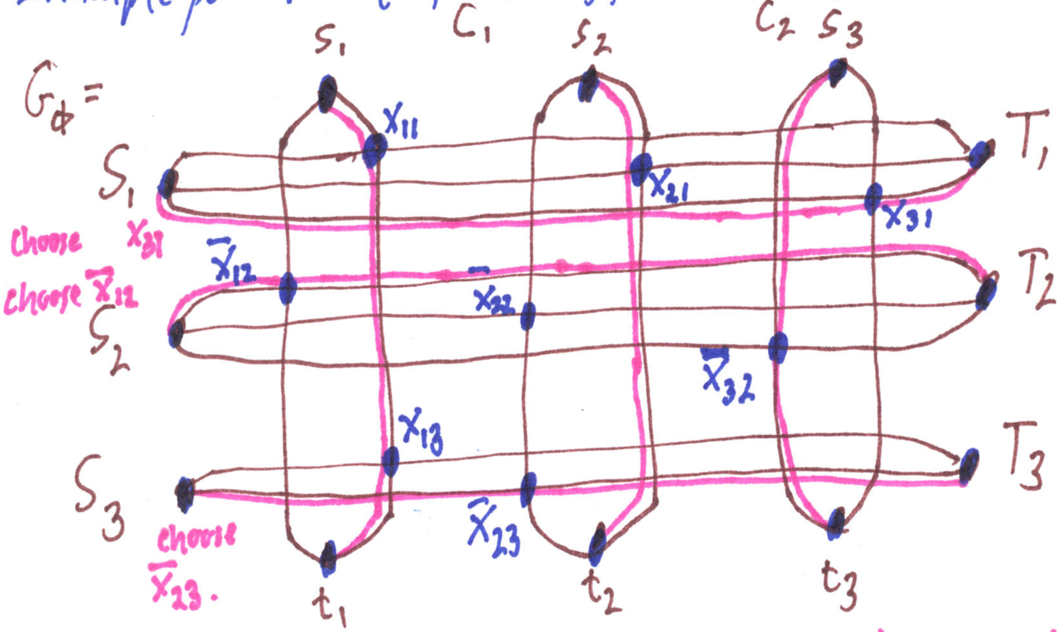
because the only possible paths conflict at the center node c.

Is it in P for $K=2$? Hmmmm... $K=3$?
What about for any fixed K? Hmmmm...

We will show it is NP-complete when K is allowed to vary. The idea is to take $K = n+m$ and dedicate n $s_i - t_i$ pairs to truth assignments and the other m pairs, capitalized as $S_j - T_j$, for the clauses C_j in ϕ.

Example for $\phi = \underbrace{(x_1 \vee x_2 \vee x_3)}_{C_1} \wedge \underbrace{(\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3)}_{C_2} \wedge \underbrace{(x_1 \vee \bar{x}_2)}_{C_3}$: Note that the "short clause" has only two horizontal options.

$G_\phi =$



The graph G_ϕ and its distinguished source and target nodes can be "streamed" quickly from the clauses of ϕ, so the reduction $f(\phi) = \langle G_\phi, \dots\rangle$ is poly-time computable.

The satisfying assignment $x_1 = 0$, $x_2 = 0$, $x_3 = 1$ corresponds to the disjoint paths shown. Note how the first and third horizontal paths are forced, but we could choose $\bar{x}_{22}$ in the middle instead.

The ALR ch 28 treatment has correctness in general but this gives the idea. So DIST C Paths is NP-complete. ⊠