

Theorem: $NTIME(t(n)) \subseteq DSPACE(O(t(n)))$. Requires $t(n) \geq n$ and $t(n)$ is "clockable".

Proof: Let any $A \in NTIME(t(n))$ be given. Then we can take an NTM N running in time $t(n)$ such that $L(N) = A$ and (essentially without loss of generality) every case of nondeterminism in N has exactly two branches: 0 -branches and 1 -branches.

Define a DTM M to work as follows: On any input x of length n , M first computes $t(n)$ and allocates $t(n)$ cells on one of its tapes to be a "tape clock".

M carries out an in-order traversal of the tree, restarts $N(x)$ using the current node y of the tree as witness or nondeterminism, and accepts x iff some y causes the simulated computation " $N(x)$ using y " to accept. Then $L(M) = A$.

(If the loop over y finishes without getting any accepting, M rejects x .)

Diagram: Computation Tree of $N(x)$
 A tree diagram showing nodes and branches. The root is labeled x_1 . The tree has a depth of $t(n)$. Nodes are labeled with 0 and 1 . The tree is labeled "Computation Tree of $N(x)$ ".

Diagram: Turing Machine M
 A diagram of a Turing Machine M with multiple tapes. The input tape contains $x_1, \dots, x_n, \square, \dots$. The "Binary Witness Counter Tape" contains $t(n)$ cells. An "extra tape to trace traversal of the full binary tree of depth $t(n)$ " is shown. Tapes holding IDs I, J, \dots of $N(x)$ are shown, one at a time. Each ID of $M(x)$ takes up space at most $|Q| + k \log t(n) + k \log t(n) = O(t(n))$ space. M has K tapes, $(K-1)$ worktapes. $DSPACE(O(t(n)))$ is indicated.

Calendar: "Recitation" Tomorrow 2-2:30 then office until 3:30

Mon Nov 11: No Class

Makeup Tue 12th, 2-3pm (Days 33)

Fri Nov 22: No Class

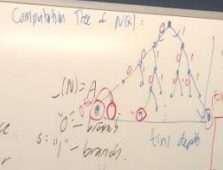
Mon Nov 25: Prelim II

Tue Dec 2: Makeup (class 2-3pm (?)).

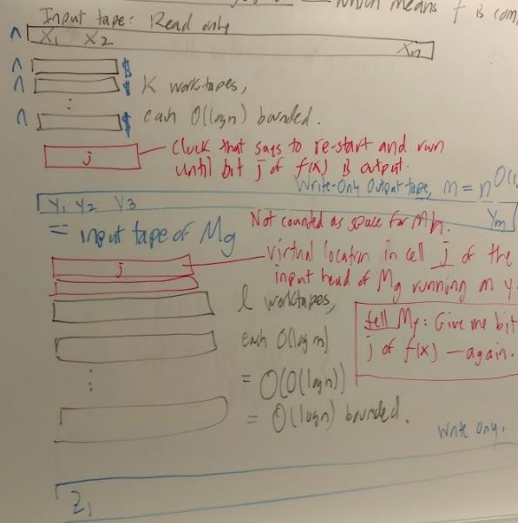
Diagram: Complexity Class Hierarchy
 A diagram showing the hierarchy of complexity classes: EXP (containing RBF), $PSPACE$, NP , $Co NP$, P , NL , L , and E . $CVP =$ Circuit Value Problem is shown between NP and $Co NP$. $DSPACE(O(n)) = NSPACE(O(n))$ is shown at the bottom. A note says "now means $A \leq \frac{\log}{m} B$ ".

Diagram: Complexity Class Relationships
 A diagram showing the relationships between complexity classes: $DTIME(O(n)) \subseteq NTIME(O(n)) \subseteq DSPACE(O(n)) \subseteq NSPACE(O(n))$.

Defⁿ: $A \leq_m^{\log} B$ if there is a function $f: \Sigma^* \rightarrow \Sigma^*$ such that $\forall x: x \in A \Leftrightarrow f(x) \in B$ and f is computable in logspace — which means f is computable by a DTM M_f . Tl;dr so:



M_f
 M_h
 M_g



Lemma: The class PL of logspace Computable functions is closed under composition: if f and g are in PL, then so is $h(x) = g(f(x))$. Hence $A \leq_m^{\log} B \wedge B \leq_m^{\log} C \Rightarrow A \leq_m^{\log} C$.

Proof: M_h as shown uses only $O(\log n)$ space, though it needs more time.

