CSE596 Lecture Mon Nov 5 Fall 2018

1) $E_{TM} \leq_m E_{2HOFA}$

III

$M \xrightarrow{f} H$

$H = f(M)$.

HW5 tips ←

2) $D_{TM} \leq_m CN^2TIME$

$M \xrightarrow{g} M'$

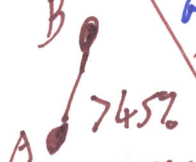
M does not acc ⟨M⟩ ⇒ . . .

M does acc ⟨M⟩ ⇒ M' ... F ... $\notin CN^2TIME$

Failed to make M' misbehave

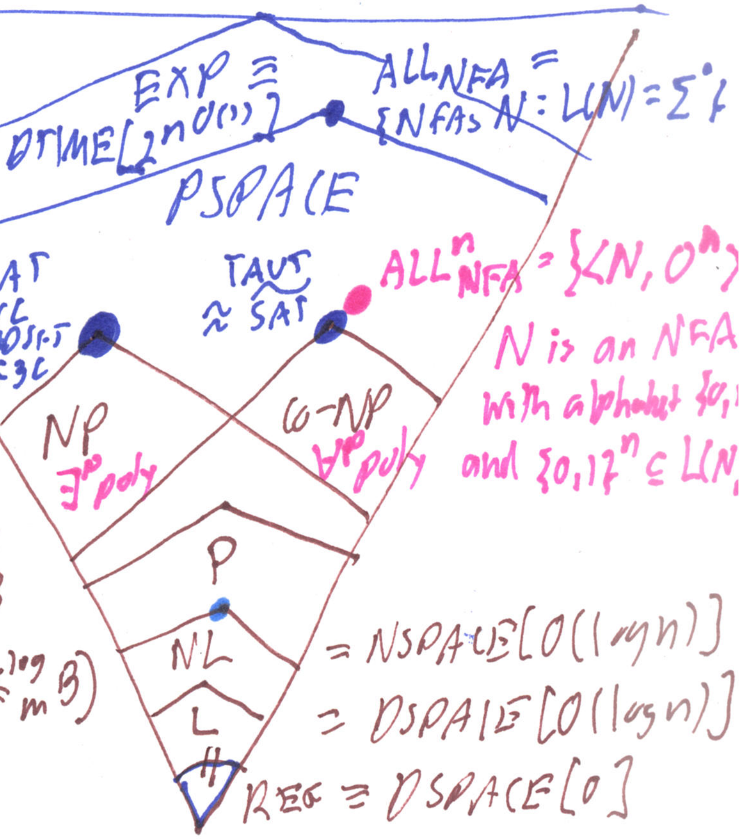
→ Lecture

NL ≠ PSPACE, P ≠ EXP are known, rest unknown.



means $A \leq_m^p B$

(later: $A \leq_m^{\log} B$)



Theorem: For any "reasonable" $s(n) \geq \log_2 n$, $t(n) \geq n+1$,

$DSPACE[s(n)] \subseteq NSPACE[s(n)] \subseteq DTIME[2^{O[s(n)]}]$ ②

↑ DTM is a NTM immediate

Point is that we have come full circle back to DSPACE but we bumped up an exponential in the middle.

$DTIME[t(n)] \subseteq NTIME[t(n)] \subseteq DSPACE[t(n)]$ ④

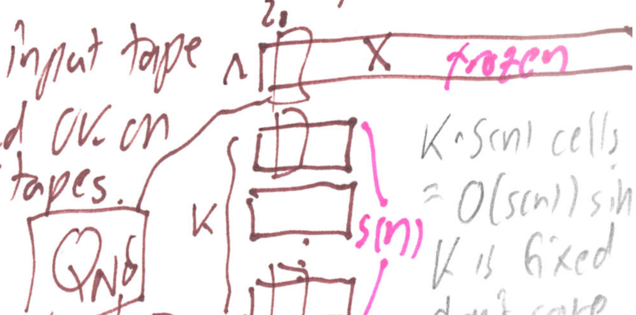
↑ immediate

Proof of Containment ②: Let any $s(n)$ space-bounded NTM N be given.

Goal: Create a $2^{O(s(n))}$-time algorithm to decide L(N). Algorithm M:

↓ input x

Wlog N has a read-only input tape and $s(n)$ cells marked off on one or more worktapes.



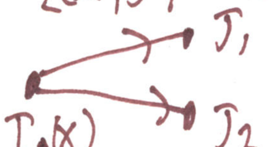
$k \cdot s(n)$ cells $= O(s(n))$ since k is fixed don't care

Build $G_x = (V, E_x)$ where

$V = \{\langle q, w, i_1, \dots, i_k \rangle$ IDs$\}$

↑ contents of worktapes, not frozen x.

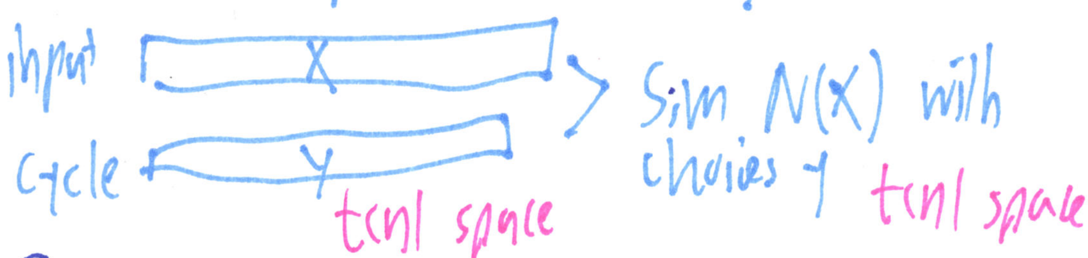
$E_x = \{(I, J) : I \vdash_N J\}$



N accepts x ⟺ G_x has a path from the start ID $I_0(x)$ to an accepting ID. Can tell this by running BFS.

How long? poly time in $\|G_x\|$ $\approx \|Q\| \cdot 2^{O(s(n))} \cdot n \cdot s(n)^{k-1}$ $\leq 2^{O(s(n))}$ time. ☒

Proof of Containment (4): Let any NTM N running in time $t(n)$ be given. Then on any input x, $|x| = n$, $N(x)$ can use at most $t(n)$ bits of nondeterminism. Hence we can cycle thru all $2^{t(n)}$ possible nondet' y; using just $t(n)$ cells to track y.



→ DTM M using $2t(n)$ space so LIN $\subseteq$ DSPACE$[O(t(n))]$.

☒

Added: The two other main "Simulation Theorems" are:

- Savitch's Theorem: $NSPACE[s(n)] \subseteq DSPACE[s(n)^2]$.

With $s(n) = \log_2 n$, an example is $NL \subseteq DSPACE[\log^2 n]$.
With $s(n) = n^k$ you get $NSPACE[n^k] \subseteq DSPACE[n^{2k}]$.
Well, for fixed k, n^{2k} is still a polynomial, so this is why "NPSPACE" $=_{def} NSPACE[n^{O(1)}]$ just equals $PSPACE =_{def} DSPACE[n^{O(1)}]$.
This will proved in-tandem with showing that QBF is PSPACE-complete.

- Immerman-Szelepcsényi Theorem: $NSPACE[s(n)] = co\text{-}NSPACE[s(n)]$

This was proved separately by Neil I. and Robert S. in 1988. Before then we had "co-NL" as a separate concept, but now we know $NL = co\text{-}NL$. Whether this has anything to say about $NP \stackrel{?}{=} coNP$ is unclear. The proof is quite hard (skip)