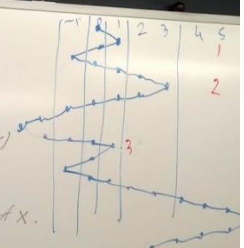
CSE596 Lecture Wed. Nov. 6, 2019

Two "Handwired" Facts:

Pippenger-Fischer 1978 after Hennie-Stearns 1966.

Theorem: Given any k-tape DTM M we can build a 2-tape DTM M' such that on any input x and for any t, $M'(x)$ simulates t steps of $M(x)$ using $O(t \log t)$ steps of its own. Moreover, the sequences of cells $c_0, c_1, c_2, \dots, c_t$ of each of the two heads of M' depends only on the length $|x|$ of x.

M' is "Oblivious"



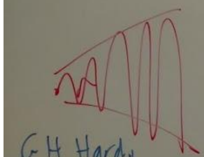
Every "jag" simulates 1 step of M and doing t jags takes $O(t \log t)$ total moves.

Tape 1 has $2k$ tracks. Tape 2 is used only to ferry data to near and away from the origin

Corollary: We can further build circuits $[C_n]_{n=1}^{\infty}$ of size $O(t(n) \log t(n))$ rather than $O(t(n)^2)$ shown in class and notes such that $\forall x \in \{0,1\}^n$, $C_n(x) = M(x)$.

Corollary 2: The reduction from any A in $NTIME(r(n))$ to 3SAT can be made to run in $O(r(n) \log r(n)) = O(r(n) \log n)$ time. since $r(n) = n^{O(1)}$

$x \sin(\frac{1}{x})$

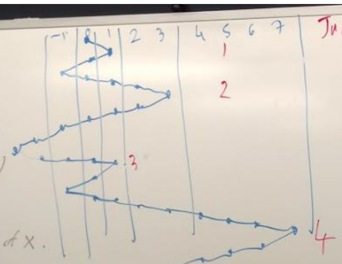


G H Hardy 1910s "folklore"

Theorem: Let $f(n)$ and $g(n)$ be any functions built up from $+, \cdot, -, /$, exp, and log. not sin or cos.

Then either • $f(n) = o(g(n))$, • $f(n) = \Theta(g(n))$, or • $g(n) = o(f(n))$. "Trichotomy of O"

Moreover, if $f(n) \geq n+1$ then f is "fully time constructible" and if $g(n) \geq \log(n)$ then g is "fully space constructible".



Jags

Deterministic Time Hierarchy Theorem:

Suppose $t_1(n) \log t_1(n) = o(t_2(n))$ with $t_2(n)$ being "fully time constructible". Then $DTIME[t_1(n)]$ is properly contained in $DTIME[t_2(n)]$

needs $t_1(n) \geq n+1$

Det^c Space Hierarchy Theorem:

If $s_1(n) = o(s_2(n))$ ~~and $s_2(n)$ is fully space constructible~~ [or ...?], then $DSPACE(s_1(n)) \subsetneq DSPACE[s_2(n)]$.

need $s_1(n) \geq \log(n)$

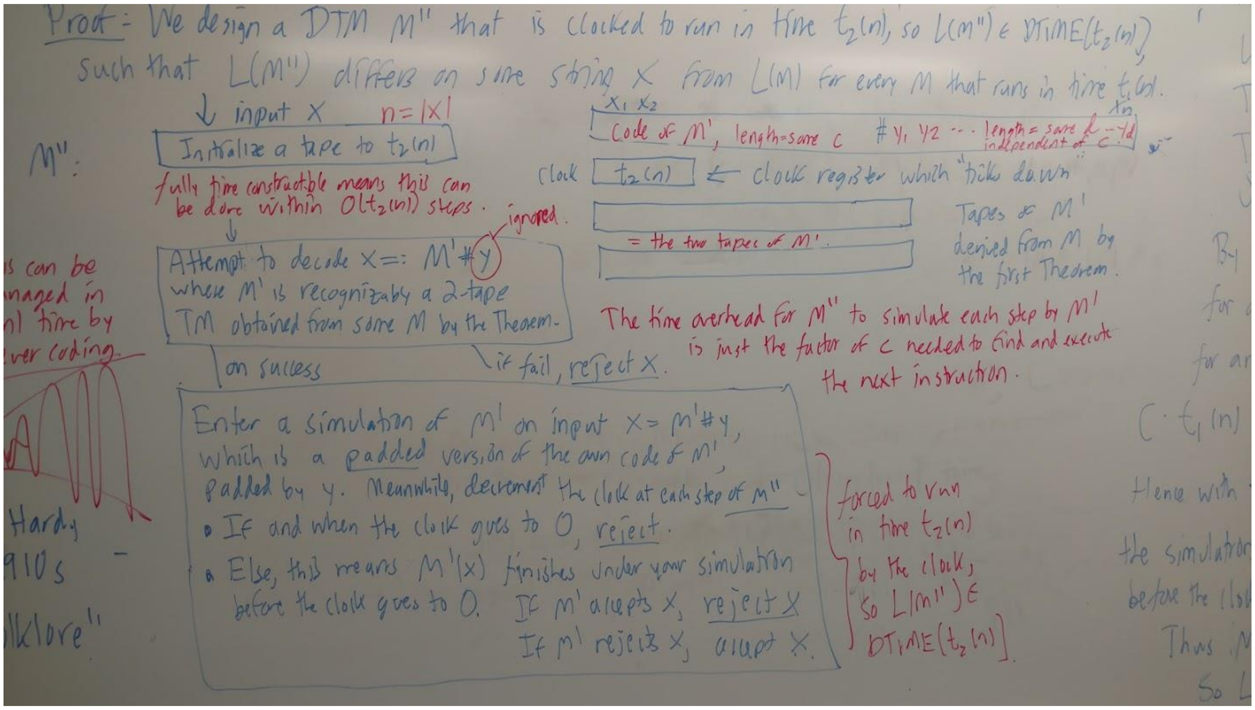
Examples:

$DTIME[O(n)] \subsetneq DTIME[O(n^2)] \subsetneq DTIME[O(n^3)] \subsetneq \dots \subseteq P$.

$\underbrace{DSPACE[O(\log n)]}_{L} \subsetneq DSPACE[O(\log n)^2]$ — note: $2^{O(\log n)^2} = n^{O(\log n)}$, not polynomial

$\subsetneq DLBA \subsetneq DSPACE[O(n^2)]$

Note: We still don't know whether $L \subsetneq P$ (or even NP!)



The cut-off part at left is the same as before and does not matter here. Next adds the end of the proof: