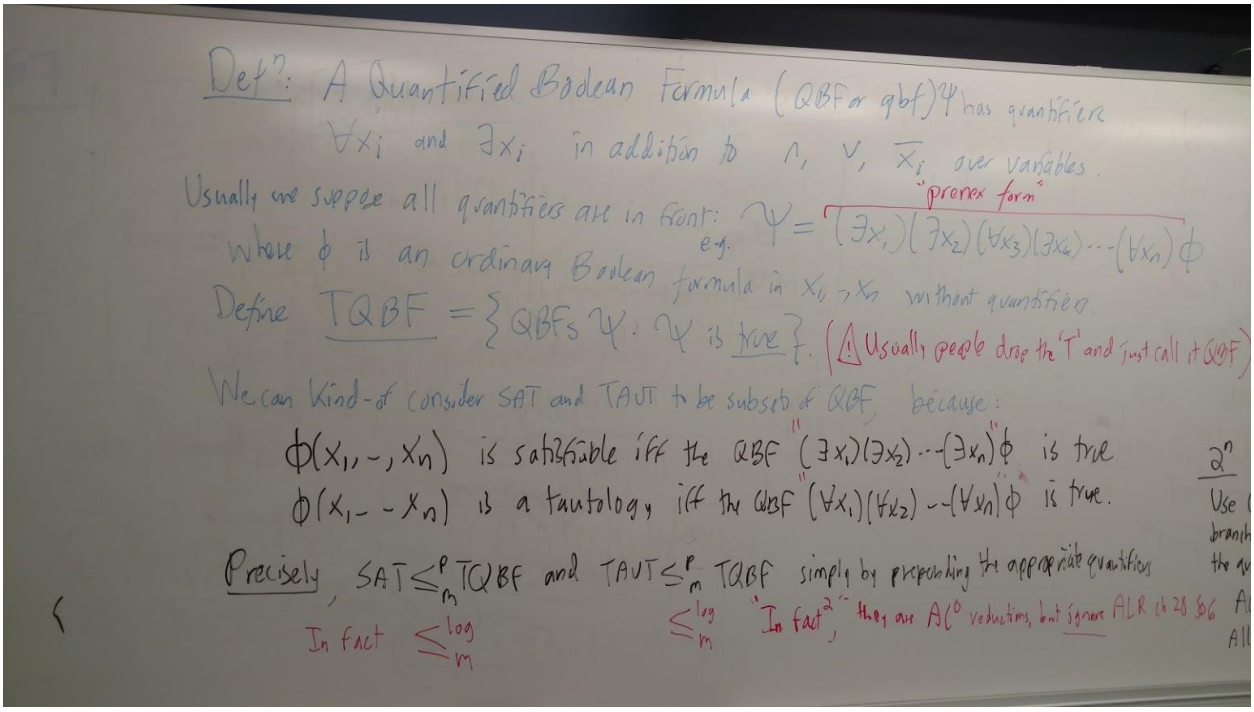
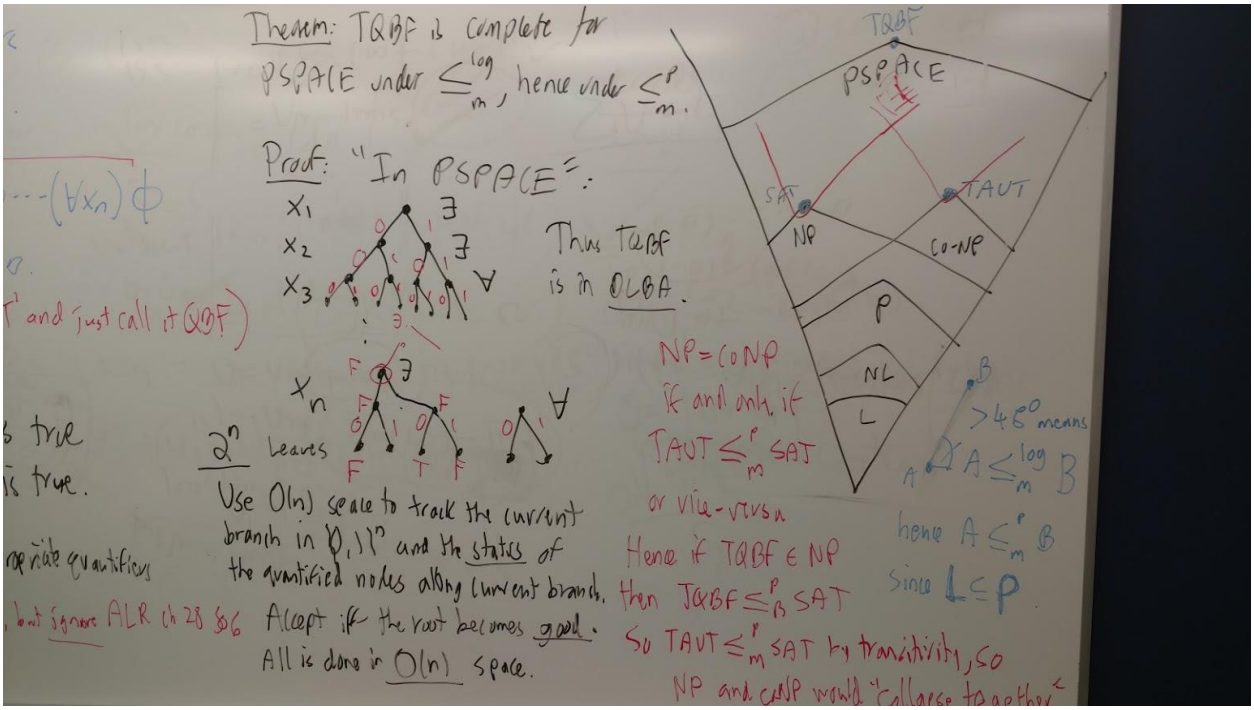
CSE596 Lecture Friday, Nov. 8, 2019





Completeness: Let any $A \in DSPACE[s(n)]$ (NSPACE) be given, where $s(n) = n^{O(1)}$. Then not only is $A \in DTIME[2^{O(s(n))}]$ but the maximum length $R(n)$ of any computation path that accepts (without cycling) is $2^{r(n)}$ where $r(n) = O(s(n))$. (so $r(n) = n^{O(1)}$.)

Idea: "A journey of a thousand miles... has a step that is exactly 500 miles from each side."

Take a NTM M such that $L(M) = A$ and on $x \in \Sigma^n$, any accepting comp^n has length at most $2^{r(n)}$ steps.

We can write a Boolean formula ϕ_0 with variables representing bits encoding IDs I, J such that an assignment to those variables satisfies ϕ_0 if and only if the corresponding IDs I, J give either $I = J$ or $I \vdash_M J$.

We write this as $\phi_0(I,J) \equiv I = J$ or $I \vdash_M J$, i.e. "I goes to J in at most 1 step".

Now for $k \geq 1$ define a formula $\phi_k(I,J) \equiv$ "I goes to J in at most 2^k steps" recursively by:

$$\phi_k(I,J) := (\exists K)\left[\phi_{k-1}(I,K) \wedge \phi_{k-1}(K,J)\right]$$

Since M runs in space $s(n)$, we may suppose $k = r$ here.

$\exists z_1 \exists z_2 \cdots \exists z_\ell$ where $z_1 \cdots z_\ell$ code an ID

Problem: The two-branch recursion blows ϕ_r up to size $2^{r(n)}$, which is too big.

$$(\exists K)(\forall I')(\forall J')\left[(I' = I \wedge J' = K) \vee (I' = K \wedge J' = J)\right] \rightarrow \phi_{k-1}(I',J')$$

$O(r(n))$

$k = r(n)$ down to 0.

Simply so we $O(\log n)$ Then $x \in$ $\circ A$

In fact NL$\subseteq$P

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$J' = J)\right] \rightarrow \phi_{k-1}(I',J')$

$k = r(n)$ down to 0.

Theorem: TQBF is complete for PSPACE under $\leq_m^{\log}$, hence under $\leq_m^P$.

Not only does this have only one recursive branch, but the recursion to produce the final output formula

$$\phi_{r(n)}(I_0(x), I_f)$$

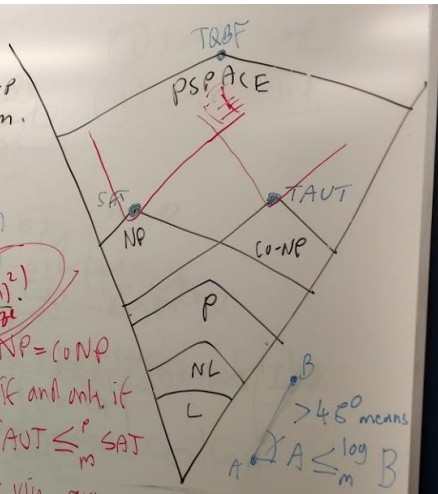
$O(r(n)^2)$ total size

Simply "n rolls" left-to-right, so we only need to keep track of $O(\log n)$-sized indices of variables.

Then $x \in A \Leftrightarrow x \in L(M) \Leftrightarrow \phi_{r(n)}$ is true.

$\circ A \leq_m^{\log}$ TQBF. $\boxtimes$

In fact, M can be an NTM, so NLOA $\leq_m^P$ TQBF in $O(n^2)$ time



NP = coNP if and only if TAUT $\leq_m^P$ SAT or vice-versa

Hence if TQBF $\in$ NP then TQBF $\leq_B^P$ SAT

So TAUT $\leq_m^P$ SAT by transitivity, so NP and coNP would "collapse together".

$A \leq_m^{\log} B$ means $A \leq_m^P B$

Since $L \subseteq P$