

We can define "randomized" analogues of other complexity classes, eg RL, BPL

This used to be a problem that could be in $RL \setminus L$.

VGAP: Inst: An undirected graph $G=(V,E)$ and $s,t \in V$.
 Ques: Is there a path from s to t ?

Special case of GAP which is complete for NL under \leq_m^{log} . For an f

Old Theorem $VGAP \in RL$
Still Imp't Theorem: If the answer is yes, then $\Pr[\text{a random walk of } t(n) \text{ steps from } s \text{ will encounter } t] > 1 - O(\frac{n^2}{t(n)})$ but the edge stream

$n = |V|$.

RL algorithm: Run randomly for $C \cdot n^2$ steps. If you see t , accept. *Certain* else reject. *unsure*. *So RL not (1-RL)*.

BUT Theorem [Omer Reingold, 2004] $RL=L$. "Derandomization of Undirected Logspace."
 Why doesn't this work for NL, i.e. for directed GAP? Consider:

Similarly, the Circuit Value Problem (CVP): Inst: A circuit C. Ques: Does $C(x) = 1$?
 Every "lozenge" is identical, so the map $\langle M, x \rangle \mapsto C_{max}$ is simple streaming.

eg RL, BPL

Special case of GAP which is complete for NL under \leq_m^{log} . For an f

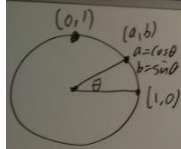
GAP \in NL: guess neighbors starting from s and accept if and when you get t as a neighbor.

For any $NSPACE[s(n)]$ -bounded TM M , $L(M) \leq_m^{log} \text{GAP}$ via the function $f(x) = G_{M,x} = (V_{M,x}, E_{M,x})$

$V = \{ \text{possible IDs } I \text{ with read-only input } x \}$
 $E_{M,x} = \{ (I, J) : I \vdash_m J \}$
 as used in the proof of $L(M) \in DTIME[2^{O(s(n))}]$ with x not needed since it is fixed.
 $|I| = \langle q, w, h_1, h_2 \rangle$

Similarly, the Circuit Value Problem is complete for P under \leq_m^{log} .
 (CVP: Inst: A circuit C with some number n of input gates, and an $x \in \{0,1\}^n$. Ques: Does $C(x) = 1$?)
 Has x not fixed

In fact, the reductions FPL: can be specified by smaller classes than FL: logspace functions.



Quantum Computing: The analogue of a char is a qubit (a,b) : $a,b \in \mathbb{C}$ and $|a|^2 + |b|^2 = 1$. A qubit is a 2-vector in the Hilbert Space \mathbb{C}^2 of norm 1. The examples $(1,0)$ and $(0,1)$ are the standard basis vectors of \mathbb{C}^2 . Since it is a basis every (other) vector is a linear combination of them.

If you pretend a, b are real numbers, you can visualize a qubit on the unit circle.

$$(a,b) = a \cdot (1,0) + b \cdot (0,1)$$

As chars, $(1,0)$ is called $|0\rangle$ and $(0,1)$ is called $|1\rangle$

How does Nature concatenate these generalized chars? $(a,b) \cdot (c,d)$

Nature does $(a,b) \otimes (c,d) = (ac, ad, bc, bd) \in \mathbb{C}^4$.

Dirac: $(a|0\rangle + b|1\rangle) \otimes (c|0\rangle + d|1\rangle) = ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle$

$\underbrace{ac}_{e_1} |00\rangle + \underbrace{ad}_{e_2} |01\rangle + \underbrace{bc}_{e_3} |10\rangle + \underbrace{bd}_{e_4} |11\rangle$

Dirac notation

Confusing? We e_1 index $(0,0)$ before $(1,0)$ but $|0\rangle$ comes before $|1\rangle$.
 but the point here is that with the edge list can be generated if streaming algo using only space

For any NSPACE[$\log n$]-B

$$f(x) = G_{M,x} =$$

Similarly, the Circuit Value Problem is

(VP): Inst: A circuit C with some number of gates.
 Ques: Does $(x) = 1$?

Every S "language" is identical, so the map $\langle M, x \rangle \mapsto C_M$ is single streaming.