CSE596, Fall 2019 **Problem Set 1** **Due Wed. Sept. 18**

Lectures and Reading. After today's lecture on regular expressions and their equivalent NFAs (with ϵ-transitions), Friday's lecture (continued into next Monday) will complete the cycle of showing those two formalisms recognize the same class of languages as DFAs. The class is standardly denoted by REG for the class of *regular* languages. Then lectures will move into the Myhill-Nerode technique for showing that certain other languages are *non*-regular and for showing that certain sets of strings must all be processed to distinguished states by a DFA. So read Debray's notes up through the end of section 3 on page 11, in tandem with the slides notes called "Extra notes for Weeks 2–3" on the course webpage (third numbered item under Required Reading). Then please also read the "Myhill-Nerode" handout (in the 5th item).

My regular office hours are Tuesdays 2–3:30pm, Wednesdays 1–1:50pm (before class). But on the 10th I am giving a presentation in CSE501 at 2pm, so I'll have hours MW 1–1:50pm instead. It is also often possible to see me right after a class.

The first two problems are based on lectures through this week; the latter two need material next week. With a weekly-HW scheme, this would be divided into two smaller sets.

(1) Let L_3 be the language of binary strings that represent *positive* multiples of 3. Define L to be the language of binary strings that *do not* have a substring in L_3. That is, L is the complement of $(0+1)^* \cdot L_3 \cdot (0+1)^*$. Follow these steps to characterize the language L.

(a) Design a DFA M_3 such that $L(M) = L_3$. It is possible that you might find this just in the course of innocently reading online notes, and that's OK, actually. (Just 3 pts., anyway.)

(b) Now make some small additions to your DFA M_3 to create an NFA N_3 such that $L(N_3) = (0+1)^* \cdot L_3 \cdot (0+1)^*$. (3 pts.)

(c) Convert N_3 into an equivalent DFA M' such that $L(M') = (0+1)^* \cdot L_3 \cdot (0+1)^*$. The sets-of-states construction to do this will be covered Friday, but you may well have had it in a previous course. (12 pts.)

(d) Complement the final states of M' to get the needed M. (3 pts.)

(e) Then answer some more short questions: First, is L infinite? Explain your answer briefly. (3 pts.)

(f) The string 11 sends M to a dead state wherever you start from because it is 3 in binary, so it makes a substring that belongs to L_3. Find a similar "dead substring" that does not have two consecutive 1s in it. (3 pts.)

(g) By "sight-reading" your machine M—or by following the formal algorithm—find a regular expression α such that $L(\alpha) = L_3$. (9 pts., for 36 total on the problem)

(2) Design both nondeterministic finite automata N_a, N_b, N_c and regular expressions r_a, r_b, r_c that denote the following three languages described in prose. (It is OK for your

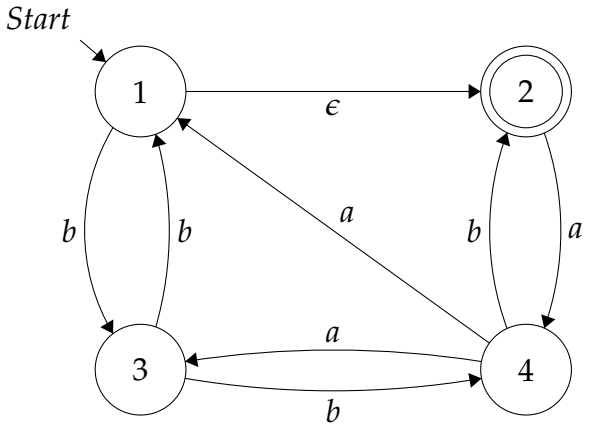
NFAs to have ϵ-arcs, and it is fine if one or more are DFAs since a DFA "Is-A" NFA.) All use the alphabet $\Sigma = \{0, 1\}$. $(3 \times (6 + 6) = 36$ points total)

(a) L_a = the set of binary strings in which the substring 10 occurs an odd number of times.

(b) L_b = the set of binary strings of the form $x = y00z$ where $|z|$ is odd.

(c) L_c = the set of binary strings having an occurrence of the substring 10 that do not have an occurrence of 11 after it.

(3) Convert the folowing NFA N into an equivalent DFA M:



Also answer (for 18 + 12 = 30 pts.):

(a) Is there a string u that N can process from its start state to each and every one of its four states? Viewing your DFA M, give a shortest u if so.

(b) Is there a string v that N cannot process from start to any state? Again give a shortest such v.

(c) Is there a string w such that no matter what string y follows it, the string wy is accepted? Again use your M.

(d) Stronger than (b) and counter to (c), is there a string z that N cannot process at all, not from any of its states p to any state q? Again use your M to explain your answer.

(4) Define $L = \{y00z : |y| = |z|\}$. Use the Myhill-Nerode technique to prove that L is not a regular language. (18 pts., for 120 total on the set)