String and Language "Borders" Examples:

The empty string is denoted by \( \varepsilon \) (or \( \lambda \) or """).
The length of \( \varepsilon \) is zero: \( |\varepsilon| = 0 \).
For any string \( x \), \( \varepsilon \cdot x = x \cdot \varepsilon = x \).

Concatenation is not commutative: \( a \cdot b \neq b \cdot a \).

The empty set \( \emptyset \) is a language, i.e. \( \text{set}\langle \text{string} \rangle \).
Its cardinality \( \|\emptyset\| \) is zero.

We want \( L(M) = \{ \varepsilon, ab, ba, abab, abba, baba, \ldots \} \).

How about \( baba \)? No (\( \varepsilon \) followed by \( 0 \) or \( a \)).

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**Definition:** An NFA is a 5-tuple \( N = (Q, \Sigma, \delta, s, F) \) where
\( Q, \Sigma, s, F \) are as in a DFA and \( \delta \subseteq (Q \times \Sigma) \times Q \).

**Typical example:**

An NFA with \( \varepsilon \)-transitions has \( \delta \subseteq (Q \times \{(\varepsilon, \varepsilon)\}) \times Q \).
Instead, which means it also has instructions \( (p, \varepsilon, q) \) which do not process a character.

Can \( (p, \varepsilon, q) \) but it has no use.

Start \( \rightarrow a \rightarrow b \rightarrow a \) NFA

\( (ab)^*(ba) \) leads to NFA

DFA

Start \( \rightarrow a \rightarrow b \rightarrow a \) leads to NFA

\( ab, abba, baba, \ldots \)?

No (\( \varepsilon \) followed by \( 0 \) or \( a \) or \( b \)).
The Wed. 9/4 lecture will pick up the definition of regular expressions and the rest of the proof of how to convert them to NFAs after first defining:

- Computations that process a string $x$ from a state $p$ to a state $q$.
- The concatenation of two languages and the Kleene star of a language.