**FA → Regex Example: Dragons & Spears DFA**

L(M) = L_{S0} \cup L_{S1}

The dead state can never be part of processing that accepts. So we can delete it. Then we have a 2-stack base case:

L_{S0} = (\alpha^* \beta \gamma)^*

L_{S1} = L_{S0} (\beta \gamma^*)^* = \alpha^* \beta \cdot \epsilon \cdot \beta^{-1} \cdot \gamma^* \cdot \beta^{-1}

Why not

L_{S1} = L_{S0} \epsilon^* \beta (0 \cup \delta)^*

Is equally correct, but redundant because \epsilon already includes trailing \alpha^*.

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**How about when we can say 2 spears? Can we "sightread" this DFA M_2?**

L_{S1} is "nice" because there is no direct reach between S and 2.

L_1 = (0 + D \alpha^* \delta + \delta (0 + \delta)^* D)^*

L(M) = L_{S0} \cup L_{S1} \cup L_{S2}

L(0^\star \delta) U O^* \cdot L_{11} U O^* \cdot L_{11} \cdot \delta (0 + \delta)^*}

Correct? Certainly sound?

Is it comprehensive?

We add the final (0^\star \delta)^*.
Definition: Given any language \( L \) (not necessarily regular), two strings \( x, y \in \Sigma^* \) are distinguishable for \( L \), written \( x \neq_L y \), if there is a string \( z \in \Sigma^* \) such that 
\[ L(xz) \neq L(yz) \]

Equation: If \( x \neq_L y \), then any DFA \( M \) such that \( L(M) = L \) must process \( x, y \) from \( s \) to different states.

\[ M(z_0) = 0, \quad M(z_1) = 1 \]

Upon starting any suffix \( z \), the processing would stay unchanged, so \( xz \) and \( yz \) would end at the same state, but \( x \) cannot be both \( a \) and \( b \).

Key Point: If \( x \neq_L y \), then any DFA \( M \) such that \( L(M) = L \) must process \( x, y \) from \( s \) to different states.

How about when we can say 2? some?

\[ M_2 \]

Mihail - Nerode Theorem, but not \( S \)

John T 1983 UB still alive, at Cornell

\[ M \]

Proof for (a): Read the key lemma, with \( k = \infty \).

\[ \text{Mihail - Nerode Theorem, but not } S \]

(a) If a language \( L \) has an infinite \( \Sigma^* \) set \( S \), then \( L \) is not regular.

(b) Moreover, every nonregular language has an infinite \( \Sigma^* \) set.

\[ \text{Mihail - Nerode Theorem, but not } S \]

Note also that \( \{a, b, \ldots, D\} \) is a \( 20 \) set of size \( 10 \), so \( M_2 \) is optimal.