Nonregularity via MNI Examples.

\[ A = \{ 0^n1^n : n \geq 1 \} \quad A = B \cap 0^+1^+ \]
\[ B = \{ x \in \{0,1\}^* : \#0(x) = \#1(x), \ x \neq \epsilon \} \]

Take \( S = 0^+ \). Clearly \( S \) is infinite. Let any \( x, y \in S, x \neq y \), be given. Then we can write \( x = 0^m, y = 0^n \) where \( m \neq n, m, n \geq 1 \). Take \( z = 1^m \). Then \( xz = 0^m1^m \in A \) (and \( \notin B \)) but \( yz = 0^n1^m \notin A \) since \( m \neq n \) (and \( \notin B \)) also.

Thus \( A(xz) \neq A(yz) \), so \( S \) is an infinite PDS for \( A \) and \( B(xz) \neq B(yz) \), "" "" for \( B \), so \( A, B \in \text{RE} \).

The same proof also goes for the complements \( \overline{A} \) and \( \overline{B} \): \( \overline{A}(xz) \neq \overline{A}(yz) \). But the Pumping Lemma is very tricky on \( B \), in particular.

\[ A' = \{ 0^n1^n : n \geq m \} \] OK if we say "Wlog \( n < m \)" in the proof.
So DFAs cannot recognize languages like A or B or "DragonsL" where you can save any # of spears?

How can we liberalize DFAs to do so?

- Allow moving L as well as R.
- Allow changing chars on the tape.

Doing just one does not increase power beyond DFA.

Doing both defines a (deterministic) Turing Machine.

Alan Turing 1936 \( \Gamma = \{0,1,\mathbf{L},\mathbf{R},_B,_,\} \)

Defn: A TM is \( M = (Q, \Sigma, \Gamma, s, s^0, s, F) \)

where \( \Gamma \) contains \( \Sigma \) and the blank and blan

\( \delta \subseteq (Q \times \Gamma) \times (\Gamma \times \{L, R, S\} \times Q) \)

Instruction \( (p, c, d, D, q) \)

M is a DTM if \( \delta \) defines a function from \( Q \times \Gamma \) to \( Q \times \Gamma \times \{L, R, S\} \).
Def: The DTM is in "nike form" if \( F = \exists q \in Q \) for one \( q \in Q \) then is another state \( q' \) with no out-arcs.
\[
\delta = Q \setminus \{ q_0, q' \} \times \Gamma \to \Gamma \times \{(L,R,L,R,S) \times Q \}
\]

Added: Prequel for Monday:

Def: A K-tape TM has the same \( M = (Q, \Sigma, \Gamma, \delta, \omega, s, F) \) but \( \delta \subseteq (Q \times \Gamma^k) \times (\Gamma^k \times \{L,R,S\}^k \times Q) \).

I like to write multi-tape tuples with chars vertical to imitate the tapes:

Typical tuple:

\[
(p, c_1 \ldots c_k, d_1 \ldots d_k, q)
\]

Graphically:

\[
K=2
\]

\[
p \quad \quad \quad (c_1/d_1/k) \quad \quad q
\]

Nice form with \( F = \exists q \in Q \) for one \( q \in Q \) is as above. Computations are harder to define but the

\[ L(M) = \{ x \in \Sigma^* : M(x) \} \text{ has a computation that halts for } x \]