Theorem: For every $K$-tape TM $M_K = (Q, \Sigma, \Gamma, \delta, q_0, s, q_f, \text{move})$, we can build a 1-tape TM $M_1$.

Proof:

1. Let $K$ be a tape on our tape $\Gamma = \{0, 1, \text{blank}\}$, including "blanked gaps" to mark where the $K$-tape heads of $M_K$ were.

2. Take $G = \Gamma \cup (\Gamma \setminus \{0, 1\})^+$. Induct on $G$ which is $\neq \emptyset$. Normally, $G'': \emptyset$.

If $M_K$ really wants to write an $a$ on tape 1, then 1, other $x$, $M_1$ writes $a'$ instead, and changes the rest $D$ to $3$.

Every REW cycle was at most $3 + s$ steps, $O(3S)$ steps, where $S \leq 2$.

Every move from $\Lambda \times 3$. Simp $s \leq n$, but $M_K$ has run for $n$ steps.

The total time cost is $O(n)$.

(Handwritten notes and diagrams for explaining the process of simulating $M_K$ on the 1-tape TM $M_1$.)

2. Begin a Read-Eval-Write (REW) cycle that always begins at a state $q$ of $M_K$ and ends at a state $q'$ of $M_K$.

3. At the begin $q_f$ of $M_1$ to the right of $\Lambda$ and all heads of $M_1$ are between $\Lambda$ and $q_f$.

4. Read while sweeping $L \to R$. The $K$-tape $c_i$-th of $M_K$ is always marked $x_i$.

5. Execute the current instruction $(q, x_i, c_i)$ in a 3-tape SWEEP form.

6. Write changes to the tape by $M_K$ using $x_i$ as read $i$ back to $\Lambda$. 

(Additional notes and diagrams for explaining the read and write process.)
Also some discussion of Assignment 1: