The lines cut off at the top said “input x \in \Sigma^*” and “yes $\rightarrow$ Reject x.”
**Dec**: A language L is said to be **decidable** if there exists a Turing machine M that halts on all inputs and accepts L if and only if the input is in L.

**Solvable**: A language L is said to be **solvable** if there exists a Turing machine M that halts on all inputs and outputs 1 if the input is in L and 0 otherwise.

**A language A **reduces** to a language B** if there exists a computable function f: \( \sum^* \rightarrow \sum^* \) such that x \in A if and only if f(x) \in B.

**Key Definition**: A language A reduces to a language B, written as A \( \leq_r B \), if there is a computable function f: \( \sum^* \rightarrow \sum^* \) such that x \in A if and only if f(x) \in B.

**Theorem**: (As implied by the diagram) If A \( \leq_r B \), then
- If B is c.e., then A is c.e.
- If B is r.e., then A is r.e.

**Proof**

1. Let M be a Turing machine that accepts B.
2. Define a new Turing machine M' as follows:
   - On input x, M' computes f(x) = f(x) = x
   - M' simulates M on input f(x).
   - Accept if M accepts.

This M' is computable and accepts A = \{x | f(x) \in B\}.

**Complexity Theory Analogy**

**Convention 1**: Classes include
- Classes include regular languages
- Classes include context-free languages
- (unless otherwise indicated)

**Convention 2**: Classes include
- Classes include context-sensitive languages
- Classes include recursively enumerable languages

**Convention 3**: Classes include
- Classes include recursive languages
- Classes include primitive recursive languages

**Complexity Classes**

- **RE**: Recursively Enumerable
- **co-RE**: Complement of RE
- **DE**: Decidable
- **co-DE**: Complement of DE

**Theorem**

If \( \text{f:} \sum^* \rightarrow \sum^* \) is a polynomial-time computable, then

**Corollary**

**Example**: Consider the function f: \( \sum^* \rightarrow \sum^* \) defined by f(x) = x^2.

1. **Example**
   - f(x) = x^2
   - So, A = B = \{x^2 \in \sum^* \}

**Exercise**

Prove that if A \( \leq_r B \) and B is r.e., then A is r.e.