Three Types of Reductions:
1. Wait for G
2. All or Nothing
3. Delay Switch

COFFEE: Instance: A TM or Java program with alphabet ASCII.

Question: Is there an input \(x \in \Sigma^*\) such that \(P(x)\) eventually writes "COFFEE" (on a tape or to std. output).

Theorem: The language \(L_{COFFEE} = \{ \langle P \rangle : \exists x \in \Sigma^* : P(x)\) outputs "COFFEE"\} is c.e. but not decidable.

Proof: "Is c.e.": take an NTM that, given \(\langle P \rangle\) as input guesses \(x\), runs \(P(x)\), and accepts if \(\bar{w}\) when \(P(x) = "COFFEE"\). We can convert the NTM \(N\) into a DTM \(M\) s.t.: \(L(M) = L(N) = L_{COFFEE} \downarrow\)

Undecidable: Show that \(\text{ATM} \leq_m L_{COFFEE}\) via the following reduction \(f:\)

\[
\langle M, w \rangle \mapsto M':
\]

This \(f\) is computable by simple code edits (adding these states for Java \(P\)) just add a line System.out.println("COFFEE") when the "Swing Kit" shows the " Simply Accepted" box.

Correctness: For all \(\langle M, w \rangle\): \(\rightarrow \) for all inputs \(x\), \(M'\) writes "COFFEE" \(L(M, w) \in \text{ATM} \rightarrow M\) accepts \(w \rightarrow \) for some \(x\), \(M'\) writes "COFFEE" \(\equiv M' \in L_{COFFEE}\)

\(L(M, w) \in \text{ATM} \equiv M\) never accepts \(w \rightarrow \) for all \(x\), \(M'\) never writes "COFFEE" \(\equiv M' \in \overline{L_{COFFEE}}\) so \(\text{ATM} \leq_m \overline{L_{COFFEE}}\) via \(f\), so \(L_{COFFEE}\) is undecidable.
General "Moral": Any inessential program behavior is undecidable.

You can write a program like Tunko Kit that never exhibits that feature a behavior.

Example Behaviors:

1. Turing Machine $M'$ makes 3 Right moves in a row.
2. $M'$ never makes L? Essential, otherwise no decidable! (on a given $x$, that is)
3. $P$ is an assembly program with numbered instructions, and the behavior is $P(x)$ JMPing to a lower-numbered instruction.

Decidable: Just run $P(x)$. Either it jumps to a lower instruction, whereupon you say "yes," or you exit over the last instruction, so answer is no. Because this is decidable, JMP back is essential.

INST: $P, x$

QUEST: Does $P(x)$ ever jump back?

Decidable "primitively"?

INSTANCE $P$ is a toy program

QUEST: Does there exist $x$ st $p(x)$ jumps back?

Is it decidable?!! Study Q.
2. All or Nothing Switch: $A TM \leq_m \forall TM$.

$(M, w) \not\in M' \Rightarrow M'$ [cf.]

Simulate $M(L_M)$

Get $K_{TM} \leq_m \forall TM$.

$(M, w) \not\in A TM \Rightarrow M$ accepts $w \Rightarrow \forall x, M'(x)$ accepts $\Rightarrow L(M') = \Sigma^*$

$(M, w) \not\in A TM \Rightarrow M$ does not accepts $w \Rightarrow \forall x, M'(x)$ never accepts $x$.

$\Rightarrow L(M') = \emptyset \Rightarrow M' \notin \forall TM$.

$\Rightarrow (M') \in \forall EM$.

Flipping around to complements, this also shows $0 TM \leq_m \forall EM$.

**ETM:**

**INST:** A TM $M$

**QUES:** Is $L(M) = \emptyset$? Complementary question to $\forall EM$.

Other things to do with $x$: e.g., accept $x$ if $x$ is a palindrome.

Other reductions and Rice's Theorem:

$(M, w) \not\in M'$ st. Wed: that and the Delay Switch. **Preview:**

$(M, w) \not\in A TM \Rightarrow \forall x, M$ accepts $x$ if $x$ is a palindrome $\Rightarrow L(M') = PALS \subseteq L(M') \not\in \emptyset$

$(M, w) \not\in A TM \Rightarrow \forall x, M$ new accepts $x \Rightarrow L(M') = \emptyset \subseteq L(M')$ is regular.

So the problem $\text{REGULAR}_{TM}$, "Given a TM $M$, is $L(M)$ regular?" is undecidable.