Theorem: A non-uniform family of one-tape, single-head Turing machines (M_i) that runs in time \( t(n) \) and space \( s(n) \) can be converted into a sequence of Boolean circuits \( C_n \) with \( n \) binary inputs each of which has \( O(\min(\log n, t(n), s(n))) \) gates, such that for all \( x \in \Sigma_n^* \), \( x \in L(M_i) \iff C_n(x) = 1 \).

Proof Outline:

- \( A \) is a language that is \( \Sigma_1 \)-complete.
- For each \( n \), let \( M_n \) be a Turing machine that runs in time \( O(\log^2 n) \) and space \( O(\log n) \) on inputs of length \( n \).
- Define \( C_n \) to be the Boolean circuit that simulates \( M_n \) on inputs of length \( n \).

It also has Boolean circuits with the same \( \log n \) width and \( O(\log^2 n) \) depth under the binary coding.

Moreover, \( C_n \) can be done in \( \Theta(n) \) time.

Theorem: We can also do

Every block of 6 cells has a Boolean circuit that enforces the same "triominos" relation we saw for PCP.

- (a) \( c, d, e \) and \( (q, c/b, R, R, r) \) are
- (b) For a 3-row, it is
- (c) For a 3-row, it is
- (d) For a 3-row, it is
- (e) For a 3-row, it is
- (f) For a 3-row, it is
- (g) For a 3-row, it is
- (h) For a 3-row, it is
- (i) For a 3-row, it is
- (j) For a 3-row, it is
- (k) For a 3-row, it is
- (l) For a 3-row, it is
- (m) For a 3-row, it is
- (n) For a 3-row, it is
- (o) For a 3-row, it is
- (p) For a 3-row, it is
- (q) For a 3-row, it is
- (r) For a 3-row, it is
- (s) For a 3-row, it is
- (t) For a 3-row, it is
- (u) For a 3-row, it is
- (v) For a 3-row, it is
- (w) For a 3-row, it is
- (x) For a 3-row, it is
- (y) For a 3-row, it is
- (z) For a 3-row, it is

By \( M \) doing "good housekeeping", the output gate of \( C_n \) can produce the 1 from the final IO (true) or the 0 from the left (false).
Skechers

Take \( n = |x| \) and build \( C_n \) for \( M \) as before.

Every gate \( \psi \in \text{NAND} \)

So consider any gate

Conjecture

For the formula \( \Phi_x \), we first allocate variables

\[ x_1, \ldots, x_n \]

To the \( n \) inputs of the gate \( \psi \)

For the \( n \) outputs\( \psi \)

For each output wire \( \psi \)

Then any assignment satisfies

The classes \( C_n \) if it makes \( \Phi_x \) true.

Finally, if \( \psi \) is a produced in

\[ O(n^2) = O(t^2) \]

polynomial time by simple translation of the gates and wires in \( C_n \).

The full circuit \( C_n \) consists of overlapping triominos in

A \( n \times n \) grid.

By M having "good housekeeping", the output gate of \( C_n \) can pluck

1 from the final \( 30 \) (once)
or

0 from \( (1, 1) \).