SAT is $\text{NP}$-complete, i.e. $\text{SAT} \in \text{NP}$ (already seen) and for all $A \in \text{NP}$, $A \leq^p_m \text{SAT}$.
[We will prove a reduction to a subcase of SAT]

Proof: By $A \in \text{NP}$, there is a polynomial $p(n)$ and a single-tape TM $M(x, t)$ that computes a witness predicate $R(x, t)$ with $|t| \leq p(|x|)$.

From Monday, we saw that $R(x, t)$ can be the computation checking predicate up to $p(n)$ tape cells; it might use in full stack.

The possible empty cells of any 6-cell
"Lozenge" can be written as a function depending only on $S$ of $M_x$.
It determines the middle bottom cell as a function of the three above.

Each lozenge has the same function $\Delta$ of six characters (3 inputs, 1 output) over $\Sigma = \{0, 1\}$.

We can re-code these over binary and write the while $\Delta$ function as a Boolean function, using NAND gates only. $\therefore M_x$ can be simulated by $p(n) \times |\Sigma|^{p(n)}$ sized circuits. (Can get $O(p(n)) \log p(n)$...
We have abstracted this to a circuit $C_n$ s.t. \[ x \in \{0, 1\} \implies \exists y, z \in \{0, 1\} \implies C_n(x, y, z) = 1 \]

\[ x_i \rightarrow \overline{x_i} \rightarrow x_i \rightarrow y_i \rightarrow \overline{y_i} \rightarrow y_i \rightarrow \cdots \rightarrow y_{p(n)} \]

- We can stamp out $C_n$ in $O(p(n))$ time.
- Gate $g$ functions correctly on bits $u, v, w$, i.e., $u \oplus v \oplus w = 1$ if and only if $\phi_g$ is made true, where

\[ \phi_g = (uvw) \land (vrvw) \land (\overline{u}rv\overline{w}) \]

- One part that depends on $n$ is $1 \times 1$
- Singleton clauses $(x_i)$ or $(\overline{x_i})$ to set those vars to the actual bit $x_i$. E.g.,

\[ x = 011 : (\overline{x_1}) \land (x_2) \land (x_3) \]

- wires $u, v, w$.

Thus $\phi_X$ is computed by an $O(p(n)^2)$ time function of $X$, and for all $x \in \Sigma^*$:

\[ x \in A \iff (\exists y, z \in \{0, 1\} \implies R(x, y, z) \land (\exists y, z \in \{0, 1\} \implies C_n(x, y, z) = 1 \iff \exists (y, w) \phi_x (x, y, w) = 1 \iff \phi_x \in SAT. \]

Observe: Every clause is at most 3 literals, all of the same sign, and $\phi_x$ is a conjunction of those clauses. 3SAT.