To finish the $PSPACE$-completenes of TAVE we initialize
\[ \overline{e}_0(I, K) \equiv I = K \lor I \downarrow \bar{K} \]
as a qbf.

\[ I = \langle q, w, \bar{h} \rangle \]

\[ \begin{array}{c}
\text{state} \\
\text{content of} \\
\text{on K tapes} \\
\text{not including} \\
\text{the input x}
\end{array} \]

\[ \text{Then becomes} \quad (x_1 = \bar{x}_1) \land \ldots \land (x_r = \bar{x}_r) \]

\[ (x_i \lor \bar{x}_i) \land (x_i \lor \bar{x}_i) \]

\[ I \downarrow \bar{K} \]

Uses two levels of the formula now look-like

Need only $O(\log(r))$ space to keep track of what similar part you are outputting.

To output the final $\overline{e}_r$, we just iterate the recursion:
\[ \overline{e}_i = (\exists J)(\forall I', J') \]

\[ \begin{array}{c}
\text{with} (I, K) \\
\text{with} (I', J')
\end{array} \]

 Allocate 3 new banks of vars
\[ I' = x'_1, \ldots, x'_r \]
\[ J' = y'_1, \ldots, y'_r \]
\[ J = y_1, \ldots, y_r \]

At the very top level, allocate/fix $K$ as $\bar{x}_K$.

Final qbf $\overline{e}_r$ has $O(r^2) = O(scn^2)$ size and is computed in $O(\log(scn))$ space. When $scn = \text{poly}(n)$, this is a $\leq \log^2$ reduction from $AExpSpace$ to $TAVE$. 

[Diagram showing the recursive process]
Moreover, brute-force solving $\exists r$ decides $A \in O(n^4) = O(s(n)^2)$ space.

... 

Facts

- GAP is complete for NL under $\leq^m$. 
- So NL = L $\iff$ GAP $\in$ L.
- GAP = \{<G, s, t>: \text{there is a path from } s \text{ to } t \text{ in the digraph } G\}.

- The Circuit Value Problem CVP
  - INST = A Boolean circuit C with n input gates and an $x \in \{0, 1\}^n$.
  - ACTS = \{s: C(x) = 1\}$
- CVP is complete for P under $\leq^m$, so CVP $\in$ L $\iff$ P = L
- CVP $\in$ NL $\iff$ P = NL.

**Proof**: Hard

**Final Fact**: NL $\neq$ co-NL, indeed for any $s(n)$-

**FACTORIZING**: As a function, if $x$ is a number, $\text{fix}(x)$ = its unique prime

**Can we do this in time $N^{O(s)}$, where $N = |x| = \Theta (\log_2 x)$?**

As a language, FACT = \{<x, w>: w is a prefix of the unique prime factorization of x \}

$\exists r \in$ FACT. As $<x, 0> \in$ FACT? $x = p_1 q_1 \cdots p_2 q_2 \cdots$.

**Depends on binary code for <, >, digits but $\text{upf}(x) = \langle p_1, q_1, p_2, q_2, \ldots \rangle$**

Anyway, getting yes/no answer let's us build up the factorization bit by bit.

**FACT \& WO TAPNP**: whether will right or wrong have the same witness of guessing $\text{upf}(x)$.