CSE681, Spring 2012 Problem Set 4 Due Wed. 4/4

Reading and Exercises:

We are now covering the Advanced notes, lectures A1–A5 as a bulk unit. Also look ahead to A6 (some of which I've previewed) and A7, but note that I may decide to do something else besides the details of division being in logspace-uniform TC^0. Also read this recent blog item

`https://rjlipton.wordpress.com/2012/03/11/a-note-on-distributions-and-approximation/`

in conjunction with the main problems on this set. Assignment 3 is now due on Monday 3/26; I will be here Wed. 3/28 but away until Wed. 4/4.

(1) Show that Parity can be recognized by uniform polynomial-size constant-depth circuits of unbounded fan-in AND/OR/NAND/NOR gates, plus $\mathsf{Mod}(m)$ gates where $m = 2^k$ is any power of 2. Does your same argument work when $m = 2k$ is any multiple of 2? Then try to show any $\mathsf{Mod}(p)$ gate can be simulated by a constant-depth poly-size network of $\mathsf{Mod}(p^k)$ or $\mathsf{Mod}(kp)$ gates, according as your answer to the case $p = 2$. Do you need p to be prime? (9+9+6 = 24 pts., somewhat open-ended)

(2) Finish the details in the post that give the counting argument for showing that depth-d circuits for Parity require super-polynomial size for $d = o(\log n)$ (exponential when d is constant), where the circuits have unbounded fan-in AND/OR/NAND/NOR gates and also $\mathsf{Mod}(p)$ gates, where p is an odd prime. (24 pts.)

(3) *Now* try to generalize the argument to show that for any single natural number q that is not a multiple of the prime p, the $\mathsf{Mod}(p)$ function cannot be computed in $\mathsf{ACC}(\mathsf{q})$.

It is considered highly unlikely that you will succeed—though if you give a deeper and more interesting reason for failure than Z_q not being a field, it will bring open-ended extra credit. In any event, do the following question instead (30 pts.):

(3′) *Now* try to generalize the argument to show that for any single natural number q that is not a multiple of the prime p, the $\mathsf{Mod}(q)$ function cannot be computed in $\mathsf{ACC}(\mathsf{p})$.

(4) Lecture Notes A1, exercises 5 and 6. (6+12 = 18 pts.)

(5) Lecture Notes A1, exercises 4. (18pts., for 114 total on the set)