

# Knowledge Representation and Reasoning Logics for Artificial Intelligence

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## 5 Summary of Part I

**Artificial Intelligence (AI):** A field of computer science and engineering concerned with the computational understanding of what is commonly called intelligent behavior, and with the creation of artifacts that exhibit such behavior.

**Knowledge Representation and Reasoning (KR or KRR):**

A subarea of Artificial Intelligence concerned with understanding, designing, and implementing ways of representing information in computers, and using that information to derive new information based on it.

KR is more concerned with belief than “knowledge”. Given that an agent (human or computer) has certain beliefs, what else is reasonable for it to believe, and how is it reasonable for it to act, regardless of whether those beliefs are true and justified.

# What is Logic?

**Logic** is the study of correct reasoning.

There are many systems of logic (logics). Each is specified by specifying:

- **Syntax:** Specifying what counts as a well-formed expression
- **Semantics:** Specifying the meaning of well-formed expressions
  - **Intensional Semantics:** Meaning relative to a Domain
  - **Extensional Semantics:** Meaning relative to a Situation
- **Proof Theory:** Defining proof/derivation, and how it can be extended.

# Relevance of Logic

Any system that consists of

- a collection of symbol structures, well-formed relative to some syntax;
- a set of procedures for adding new structures to that collection based on what's already in there.

is a logic.

But:

Do the symbol structures have a consistent semantics?

Are the procedures sound relative to that semantics?

Soundness is the essence of “correct reasoning.”

# KR and Logic

Given that a Knowledge Base is represented in a language with a well-defined syntax, a well-defined semantics, and that reasoning over it is a well-defined procedure, a KR system is a logic.

KR research can be seen as a search for the best logic to capture human-level reasoning.

# Relations Among Inference Problems

**Syntax**

**Derivation**

**Theoremhood**

$$A_1, \dots, A_n \vdash Q \quad \Leftrightarrow \quad \vdash A_1 \wedge \dots \wedge A_n \Rightarrow Q$$

$\Downarrow \Uparrow$

$\Downarrow \Uparrow$

$$A_1, \dots, A_n \models Q \quad \Leftrightarrow \quad \models A_1 \wedge \dots \wedge A_n \Rightarrow Q$$

**Semantics**    **Logical Entailment**

**Validity**

( $\Downarrow$  *Soundness*)

( $\Uparrow$  *Completeness*)

# Inference/Reasoning Methods

Given a KB/set of assumptions  $\mathcal{A}$  and a query  $Q$ :

- Model Finding
  - Direct: Find satisfying models of  $\mathcal{A}$ , see if  $Q$  is True in all of them.
  - Refutation: Find if  $\mathcal{A} \cup \{\neg Q\}$  is unsatisfiable.
- Natural Deduction
  - Direct: Find if  $\mathcal{A} \vdash Q$ .
- Resolution
  - Direct: Find if  $\mathcal{A} \vdash Q$  (incomplete).
  - Refutation: Find if  $\bigwedge \mathcal{A} \wedge \neg Q$  is inconsistent.

# Classes of Logics

- Propositional Logic
  - Finite number of atomic propositions and models.
  - Model finding and resolution are decision procedures.
- Finite-Model Predicate Logic
  - Finite number of terms, atomic formulae, and models.
  - Reducible to propositional logic.
  - Model finding and resolution are decision procedures.
- First-Order Logic
  - Infinite number of terms, atomic formulae, and models.
  - Not reducible to propositional logic.
  - There are no decision procedures.
  - Resolution plus factoring is refutation complete.

# Logics We Studied

1. Standard Propositional Logic
2. Clause Form Propositional Logic
3. Standard Finite-Model Predicate Logic
4. Clause Form Finite-Model Predicate Logic
5. Standard First-Order Predicate Logic
6. Clause Form First-Order Predicate Logic

# Proof Procedures We Studied

1. Direct model finding: truth tables, `decreasoner`, `relsat` (complete search) `walksat`, `gsat` (stochastic search)
2. Semantic tableaux (model-finding refutation)
3. Wang algorithm (model-finding refutation), `wang`
4. Hilbert-style axiomatic (direct), *brief*
5. Fitch-style natural deduction (direct)
6. Resolution (refutation), `prover`, `SNARK`

# Utility Notions and Techniques

1. Material implication
2. Possible properties of connectives  
commutative, associative, idempotent
3. Possible properties of well-formed expressions  
free, bound variables  
open, closed, ground expressions
4. Possible semantic properties of wffs  
contradictory, satisfiable, contingent, valid
5. Possible properties of proof procedures  
sound, consistent, complete,  
decision procedure, semi-decision procedure

# More Utility Notions and Techniques

## 5. Substitutions

application, composition

## 6. Unification

most general common instance (mgci),  
most general unifier (mgu)

## 7. Translation from standard form to clause form

Conjunctive Normal Form (CNF),  
Skolem functions/constants

## 8. Resolution Strategies

subsumption, unit preference, set of support

## 9. The Answer Literal

# Unification

- Unification is a least-commitment method of choosing a substitution for Universal Instantiation ( $\forall E$ ).
- Unification is correct pattern matching when both structures have variables.
- Unification is generally needed in backward chaining systems.

# AI-Logic Connections

AI	Logic
rules or domain rules	non-atomic assumptions or non-logical axioms
inference engine procedures	rules of inference
knowledge base	derivation