

Knowledge Representation and Reasoning Logics for Artificial Intelligence

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6 Prolog

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6.1 Horn Clauses

A Horn Clause is a clause with at most one positive literal.

Either $\{\neg Q_1(\bar{x}), \dots, \neg Q_n(\bar{x})\}$ (negative Horn clause)

or $\{C(\bar{x})\}$ (fact or positive or definite Horn clause)

or $\{\neg A_1(\bar{x}), \dots, \neg A_n(\bar{x}), C(\bar{x})\}$ (positive or definite Horn clause)

which is the same as

$$A_1(\bar{x}) \wedge \dots \wedge A_n(\bar{x}) \Rightarrow C(\bar{x})$$

where $A_i(\bar{x})$, $C(\bar{x})$, and $Q(\bar{x})$ are atoms.

SLD Resolution

Selected literals, Linear pattern, over Definite clauses

SLD derivation of clause c from set of clauses S is

$$c_1, \dots, c_n = c$$

$$\text{s.t. } c_1 \in S$$

and c_{i+1} is resolvent of c_i and a clause in S . [B&L, p. 87]

If S is a set of Horn clauses,

then there is a resolution derivation of $\{\}$ from S

iff there is an SLD derivation of $\{\}$ from S .

SLDSolve

```
procedure SLDSolve(KB,query) returns true or false {  
  /* KB = {rule1, ..., rulen}  
  * rulei = {hi, ¬bi1, ..., ¬biki}  
  * query = {¬q1, ..., ¬qm} */  
  if (m = 0) return true;  
  for i := 1 to n {  
    if((μ := Unify(q1, hi)) ≠ FAIL  
      and SLDSolve(KB, {¬bi1μ, ..., ¬bikiμ, ¬q2μ, ..., ¬qmμ})) {  
      return true;  
    }  
  }  
  return false;  
}
```

Where h_i , b_{ij} , and q_i are atomic formulae.

See B&L, p. 92

6.2 Prolog

Example Prolog Interaction

```
<timberlake:~/xemacs:1:35> sicstus
SICStus 4.0.5 (x86_64-linux-glibc2.3): Thu Feb 12 09:48:30 CET 2009
Licensed to SP4cse.buffalo.edu
| ?- consult(user).
% consulting user...
| driver(X) :- drives(X,_).
| passenger(Y) :- drives(_,Y).
| drives(betty,tom).
|
% consulted user in module user, 0 msec 1200 bytes
yes
| ?- driver(X), passenger(Y).
X = betty,
Y = tom ?
yes
| ?- halt.
```


Prolog Program with Two Answers

% From Rich & Knight, 2nd Edition (1991) p. 192.

likesToEat(X,Y) :- cat(X), fish(Y).

cat(X) :- calico(X).

fish(X) :- tuna(X).

tuna(charlie).

tuna(herb).

calico(puss).

Listing the Fish Program

```
| ?- listing.  
calico(puss).
```

```
cat(A) :-  
    calico(A).
```

```
fish(A) :-  
    tuna(A).
```

```
likesToEat(A, B) :-  
    cat(A),  
    fish(B).
```

```
tuna(charlie).  
tuna(herb).
```

yes

Note: `consult(File)` loads the `File` in interpreted mode, whereas `[File]` loads the `File` in compiled mode. `listing` is only possible in interpreted mode.

Running the Fish Program

```
<timberlake:CSE563:1:39> sicstus
SICStus 4.0.5 (x86_64-linux-glibc2.3): Thu Feb 12 09:48:30 CET 2009
Licensed to SP4cse.buffalo.edu
| ?- ['fish.prolog'].
% compiling /projects/shapiro/CSE563/fish.prolog...
% compiled /projects/shapiro/CSE563/fish.prolog in module user, 0 msec 1808 bytes
yes

| ?- likesToEat(puss,X).
X = charlie ? ;
X = herb ? ;
no

| ?- halt.
<timberlake:CSE563:1:40>
```

Tracing the Fish Program

```
| ?- ['fish.prolog'].  
% consulting /projects/shapiro/CSE563/fish.prolog...  
% consulted /projects/shapiro/CSE563/fish.prolog in module user, 0 :  
yes  
  
| ?- trace.  
% The debugger will first creep -- showing everything (trace)  
yes  
% trace
```

Tracing First Answer

```
| ?- likesToEat(puss,X).  
      1      1 Call: likesToEat(puss,_442) ?  
      2      2 Call: cat(puss) ?  
      3      3 Call: calico(puss) ?  
      3      3 Exit: calico(puss) ?  
      2      2 Exit: cat(puss) ?  
      4      2 Call: fish(_442) ?  
      5      3 Call: tuna(_442) ?  
?      5      3 Exit: tuna(charlie) ?  
?      4      2 Exit: fish(charlie) ?  
?      1      1 Exit: likesToEat(puss,charlie) ?  
X = charlie ? ;
```

Tracing the Second Answer

```
X = charlie ? ;
    1      1 Redo: likesToEat(puss,charlie) ?
    4      2 Redo: fish(charlie) ?
    5      3 Redo: tuna(charlie) ?
    5      3 Exit: tuna(herb) ?
    4      2 Exit: fish(herb) ?
    1      1 Exit: likesToEat(puss,herb) ?
```

```
X = herb ? ;
no
% trace
| ?- notrace.
% The debugger is switched off
yes
```

Backtracking Example

Program:

```
bird(tweety).  
bird(oscar).  
bird(X) :- feathered(X).  
feathered(maggie).  
large(oscar).  
ostrich(X) :- bird(X), large(X).
```

Run (No backtracking needed):

```
| ?- ostrich(oscar).  
      1      1 Call: ostrich(oscar) ?  
      2      2 Call: bird(oscar) ?  
?      2      2 Exit: bird(oscar) ?  
      3      2 Call: large(oscar) ?  
      3      2 Exit: large(oscar) ?  
?      1      1 Exit: ostrich(oscar) ?  
yes
```

Backtracking Used

```
| ?- ostrich(X).  
      1      1 Call: ostrich(_368) ?  
      2      2 Call: bird(_368) ?  
?      2      2 Exit: bird(tweety) ?  
      3      2 Call: large(tweety) ?  
      3      2 Fail: large(tweety) ?  
      2      2 Redo: bird(tweety) ?  
?      2      2 Exit: bird(oscar) ?  
      4      2 Call: large(oscar) ?  
      4      2 Exit: large(oscar) ?  
?      1      1 Exit: ostrich(oscar) ?  
X = oscar ?  
yes
```


Backtracking: Effect of Query

```
/projects/shapiro/CSE563/Examples/Prolog/backtrack.prolog:
```

```
supervisorOf(X,Y) :- managerOf(X,Z), departmentOf(Y,Z).
```

```
managerOf(jones,accountingDepartment).
```

```
managerOf(smith,itDepartment).
```

```
departmentOf(kelly,accountingDepartment).
```

```
departmentOf(brown,itDepartment).
```

Backtracking not needed:

```
| ?- supervisorOf(smith,X).
```

```
1      1 Call: supervisorOf(smith,_380) ?
```

```
2      2 Call: managerOf(smith,_772) ?
```

```
2      2 Exit: managerOf(smith,itDepartment) ?
```

```
3      2 Call: departmentOf(_380,itDepartment) ?
```

```
3      2 Exit: departmentOf(brown,itDepartment) ?
```

```
1      1 Exit: supervisorOf(smith,brown) ?
```

```
X = brown ?
```

```
yes
```

Backtracking Example, part 2

```
supervisorOf(X,Y) :- managerOf(X,Z), departmentOf(Y,Z).
managerOf(jones,accountingDepartment).
managerOf(smith,itDepartment).
departmentOf(kelly,accountingDepartment).
departmentOf(brown,itDepartment).
```

```
| ?- supervisorOf(X,brown).
```

```
      1      1 Call: supervisorOf(_368,brown) ?
      2      2 Call: managerOf(_368,_772) ?
?      2      2 Exit: managerOf(jones,accountingDepartment) ?
      3      2 Call: departmentOf(brown,accountingDepartment) ?
      3      2 Fail: departmentOf(brown,accountingDepartment) ?
      2      2 Redo: managerOf(jones,accountingDepartment) ?
      2      2 Exit: managerOf(smith,itDepartment) ?
      4      2 Call: departmentOf(brown,itDepartment) ?
      4      2 Exit: departmentOf(brown,itDepartment) ?
      1      1 Exit: supervisorOf(smith,brown) ?
```

```
X = smith ?
```

```
yes
```

Negation by Failure & The Closed World Assumption

```
| ?- [user].  
% consulting user...  
| manager(jones, itSection).  
| manager(smith, accountingSection).  
|  
% consulted user in module user, 0 msec 416 bytes  
yes  
| ?- manager(smith, itSection).  
no  
| ?- manager(kelly, accountingSection).  
no
```

Negation by failure: “no” means didn’t succeed.

CWA: If it’s not in the KB, it’s not true.

Cut: Preventing Backtracking

KB Without Cut

```
| ?- consult(user).  
% consulting user...  
| bird(oscar).  
| bird(tweety).  
| bird(X) :- feathered(X).  
| feathered(maggie).  
| large(oscar).  
| ostrich(X) :- bird(X), large(X).  
|  
% consulted user in module user, 0 msec 1120 bytes  
yes
```

No Backtracking Needed

```
| ?- trace.  
% The debugger will first creep -- showing everything (trace)  
yes  
% trace  
| ?- ostrich(oscar).  
          1      1 Call: ostrich(oscar) ?  
          2      2 Call: bird(oscar) ?  
?         2      2 Exit: bird(oscar) ?  
          3      2 Call: large(oscar) ?  
          3      2 Exit: large(oscar) ?  
?         1      1 Exit: ostrich(oscar) ?  
yes  
% trace
```

Unwanted Backtracking

```
| ?- ostrich(tweety).  
      1      1 Call: ostrich(tweety) ?  
      2      2 Call: bird(tweety) ?  
?     2      2 Exit: bird(tweety) ?  
      3      2 Call: large(tweety) ?  
      3      2 Fail: large(tweety) ?  
      2      2 Redo: bird(tweety) ?  
      4      3 Call: feathered(tweety) ?  
      4      3 Fail: feathered(tweety) ?  
      2      2 Fail: bird(tweety) ?  
      1      1 Fail: ostrich(tweety) ?
```

no

No need to try to solve `bird(tweety)` another way.

KB With Cut

```
| ?- consult(user).  
% consulting user...  
| bird(oscar).  
| bird(tweety).  
| bird(X) :- feathered(X).  
| feathered(maggie).  
| large(oscar).  
| ostrich(X) :- bird(X), !, large(X).  
|  
% consulted user in module user, 0 msec -40 bytes  
yes  
% trace
```

No Extra Backtracking

```
| ?- ostrich(tweety).  
      1      1 Call: ostrich(tweety) ?  
      2      2 Call: bird(tweety) ?  
?     2      2 Exit: bird(tweety) ?  
      3      2 Call: large(tweety) ?  
      3      2 Fail: large(tweety) ?  
      1      1 Fail: ostrich(tweety) ?  
  
no  
% trace
```


fail: Forcing Failure

If something is a canary, it is not a penguin.

```
| ?- consult(user).
% consulting user...
| penguin(X) :- canary(X), !, fail.
| canary(tweety).
|
% consulted user in module user, 0 msec 416 bytes
yes
% trace
| ?- penguin(tweety).
      1      1 Call: penguin(tweety) ?
      2      2 Call: canary(tweety) ?
      2      2 Exit: canary(tweety) ?
      1      1 Fail: penguin(tweety) ?
no
% trace
```

Cut Fails the Head Instance: Program

```
penguin(X) :- canary(X), !, fail.  
penguin(X) :- bird(X), swims(X).
```

```
canary(tweety).  
bird(willy).  
swims(willy).
```

Cut Fails the Head Instance: Run

```
| ?- penguin(willy).  
    1      1 Call: penguin(willy) ?  
    2      2 Call: canary(willy) ?  
    2      2 Fail: canary(willy) ?  
    3      2 Call: bird(willy) ?  
    3      2 Exit: bird(willy) ?  
    4      2 Call: swims(willy) ?  
    4      2 Exit: swims(willy) ?  
    1      1 Exit: penguin(willy) ?  
  
yes  
% trace  
| ?- penguin(tweety).  
    1      1 Call: penguin(tweety) ?  
    2      2 Call: canary(tweety) ?  
    2      2 Exit: canary(tweety) ?  
    1      1 Fail: penguin(tweety) ?  
  
no
```

Cut Fails Head Alternatives

```
| ?- penguin(X).  
    1      1 Call: penguin(_368) ?  
    2      2 Call: canary(_368) ?  
    2      2 Exit: canary(tweety) ?  
    1      1 Fail: penguin(_368) ?
```

no

Moral:

Use cut when seeing if a ground atom is satisfied (T/F question),
but not when generating satisfying instances (wh questions).

Bad Rule Order

```
penguin(X) :- bird(X), swims(X).
penguin(X) :- canary(X), !, fail.
bird(X) :- canary(X).
canary(tweety).

% trace
| ?- penguin(tweety).
    1      1 Call: penguin(tweety) ?
    2      2 Call: bird(tweety) ?
    3      3 Call: canary(tweety) ?
    3      3 Exit: canary(tweety) ?
    2      2 Exit: bird(tweety) ?
    4      2 Call: swims(tweety) ?
    4      2 Fail: swims(tweety) ?
    5      2 Call: canary(tweety) ?
    5      2 Exit: canary(tweety) ?
    1      1 Fail: penguin(tweety) ?

no
```

Good Rule Order

```
penguin(X) :- canary(X), !, fail.  
penguin(X) :- bird(X), swims(X).  
bird(X) :- canary(X).  
canary(tweety).  
  
% trace  
| ?- penguin(tweety).  
      1      1 Call: penguin(tweety) ?  
      2      2 Call: canary(tweety) ?  
      2      2 Exit: canary(tweety) ?  
      1      1 Fail: penguin(tweety) ?  
  
no
```

SICSTUS Allows “or” In Body.

```
bird(willy).
swims(willy).
canary(tweety).
penguin(X) :-
    canary(X), !, fail;
    bird(X), swims(X).
bird(X) :- canary(X).

| ?- ['twoRuleCutOr.prolog'].
% compiling /projects/shapiro/CSE563/twoRuleCutOr.prolog...
* clauses for user:bird/1 are not together
* Approximate lines: 8-10, file: '/projects/shapiro/CSE563/twoRuleCutOr.prolog'
% compiled /projects/shapiro/CSE563/twoRuleCutOr.prolog in module user, 0 msec 928 bytes
yes
| ?- penguin(willy).
yes
| ?- penguin(tweety).
no
```

not: “Negated” Antecedents

A bird that is not a canary is a penguin.

```
| penguin(X) :- bird(X), !, \+canary(X).  
| bird(opus).  
| canary(tweety).  
% compiled user in module user, 0 msec 512 bytes
```

```
| ?- penguin(opus).  
      1      1 Call: penguin(opus) ?  
      2      2 Call: bird(opus) ?  
      2      2 Exit: bird(opus) ?  
      3      2 Call: canary(opus) ?  
      3      2 Fail: canary(opus) ?  
      1      1 Exit: penguin(opus) ?
```

yes

\+ is SICStus Prolog’s version of not.

It is negation by failure, not logical negation.

Can Use Functions

```
driver(X) :- drives(X,_).  
drives(mother(X),X) :- schoolchild(X).  
schoolchild(betty).  
schoolchild(tom).
```

```
| ?- driver(X).  
X = mother(betty) ? ;  
X = mother(tom) ? ;  
no
```

Infinitely Growing Terms

```
driver(X) :- drives(X,_).
drives(mother(X),X) :- commuter(X).
commuter(betty).
commuter(tom).
commuter(mother(X)) :- commuter(X).
```

```
| ?- driver(X).
X = mother(betty) ? ;
X = mother(tom) ? ;
X = mother(mother(betty)) ? ;
X = mother(mother(tom)) ? ;
X = mother(mother(mother(betty))) ? ;
X = mother(mother(mother(tom))) ?
yes
```

Prolog Does Not Do the Occurs Check

```
<pollux:CSE563:2:31> sicstus
```

```
...
```

```
| ?- [user].
```

```
% consulting user...
```

```
| mother(motherOf(X), X).
```

```
|
```

```
% consulted user in module user, 0 msec 248 bytes
```

```
yes
```

```
| ?- mother(Y, Y).
```

```
Y = motherOf(motherOf(motherOf(motherOf(motherOf(motherOf(
    motherOf(motherOf(motherOf(motherOf(...)))))))))) ?
```

```
yes
```

```
| ?-
```

“=” and “is”

```
| ?- p(X, b, f(c,Y)) = p(a, U, f(V,U)).  
U = b,  
V = c,  
X = a,  
Y = b ?  
yes  
| ?- X is 2*(3+6).  
X = 18 ?  
yes  
| ?- X = 2*(3+6).  
X = 2*(3+6) ?  
yes  
| ?- X is 2*(3+6), Y is X/3.  
X = 18,  
Y = 6.0 ?  
yes  
| ?- Y is X/3, X is 2*(3+6).  
! Instantiation error in argument 2 of is/2  
! goal:  _76 is _73/3
```

Avoid Left Recursive Rules

To define `ancestor` as the transitive closure of `parent`.

The base case: `ancestor(X,Y) :- parent(X,Y).`

Three possible recursive cases:

1. `ancestor(X,Y) :- parent(X,Z), ancestor(Z,Y).`
2. `ancestor(X,Y) :- ancestor(X,Z), parent(Z,Y).`
3. `ancestor(X,Y) :- ancestor(X,Z), ancestor(Z,Y).`

Versions (2) and (3) will cause infinite loops.