

Knowledge Representation and Reasoning

Logics for Artificial Intelligence

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7 A Potpourri of Subdomains

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Taxonomies: Categories as Intensional Sets

In mathematics, a set is defined by its members.

This is an **extensional** set.

Plato: *Man is a featherless biped.*

An **intensional** set is defined by properties.

Aristotle: *Man is a rational animal.*

A category (type, class) is an intensional set.

Taxonomies: Need for Two Relations

With sets, there's a difference between

set membership, \in $5 \in \{1, 3, 5, 7, 9\}$

and subset, \subset, \subseteq $\{1, 3, 5, 7, 9\} \subset \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

One difference is that subset is transitive:

$\{1, 3, 5\} \subset \{1, 3, 5, 7, 9\}$ and $\{1, 3, 5, 7, 9\} \subset \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

and $\{1, 3, 5\} \subset \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

membership is not:

$5 \in \{1, 3, 5, 7, 9\}$ and $\{1, 3, 5, 7, 9\} \in \{\{1, 3, 5, 7, 9\}, \{2, 4, 6, 8\}\}$

but $5 \notin \{\{1, 3, 5, 7, 9\}, \{2, 4, 6, 8\}\}$

Similarly, we need both the instance relation and the subcategory relation.

Taxonomies: Categories as Unary Predicates

One way to represent taxonomies:

Canary(Tweety)

$\forall x[(\text{Canary}(x) \Rightarrow \text{Bird}(x)]$

$\forall x[(\text{Bird}(x) \Rightarrow \text{Vertebrate}(x)]$

$\forall x[(\text{Vertebrate}(x) \Rightarrow \text{Chordate}(x)]$

$\forall x[(\text{Chordate}(x) \Rightarrow \text{Animal}(x)]$

Taxonomies: Reifying

To reify: to make a thing of.

Allows discussion of “predicates” in FOL.

Membership: *Member* or *Instance* or *Isa*

Isa(Tweety, Canary)

Subcategory: *Subclass* or *Ako* (sometimes, even, *Isa*)

Ako(Canary, Bird)

Ako(Bird, Vertebrate)

Ako(Vertebrate, Chordate)

Ako(Chordate, Animal)

Axioms:

$$\forall x \forall c_1 \forall c_2 [Isa(x, c_1) \wedge Ako(c_1, c_2) \Rightarrow Isa(x, c_2)]$$
$$\forall c_1 \forall c_2 \forall c_3 [Ako(c_1, c_2) \wedge Ako(c_2, c_3) \Rightarrow Ako(c_1, c_3)]$$

Discussing Categories

Isa(Canary, Species)

Isa(Bird, Class)

Isa(Chordate, Phylum)

Isa(Animal, Kingdom)

Extinct(Dinosaur)

Note: That's *Isa*, not *Ako*.

If categories are predicates, requires second-order logic.

Other relationships: exhaustive subcategories, disjoint categories, partitions.

DAG (directed acyclic graph), rather than just a tree.

E.g., human: man vs. woman; child vs. adult vs. senior.

Transitive Closure

It's sometimes useful (especially in Prolog)
to have a second relation, R_2
be the transitive closure of a relation, R_1 .

$$\begin{aligned} \forall R_1, R_2 [\text{transitiveClosureOf}(R_2, R_1) \\ \Leftrightarrow [\forall x, y (R_1(x, y) \Rightarrow R_2(x, y)) \\ \quad \wedge \forall x, y, z [R_1(x, y) \wedge R_2(y, z) \Rightarrow R_2(x, z)]]] \end{aligned}$$

E.g. *ancestor* is the transitive closure of *parent*:

$$\begin{aligned} \forall x, y [\text{parent}(x, y) \Rightarrow \text{ancestor}(x, y)] \\ \forall x, y, z [\text{parent}(x, y) \wedge \text{ancestor}(y, z) \Rightarrow \text{ancestor}(x, z)] \end{aligned}$$

7.2 Time

How would you represent time?

Discuss

Subjective *vs.* Objective: Subjective

Make now an individual in the domain.

Include other times relative to now.

OK if time doesn't move.

Subjective *vs.* Objective: Objective

Make `now` a meta-logical variable with some time-denoting term as value.

Relate times to each other, *e.g.* $\text{Before}(t1, t2)$.

Now can move by giving `now` a new value.

Points *vs.* Intervals: Points

Use numbers: integers, rationals, reals?

Computer reals aren't really dense.

How to assign numbers to times?

Granularity: How big, numerically, is a day, or any other interval of time?

If an interval is defined as a pair of points, which interval is the midpoint in, if one interval immediately follows another?

Points *vs.* Intervals: Intervals

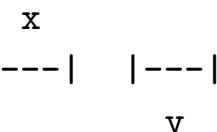
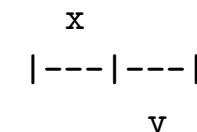
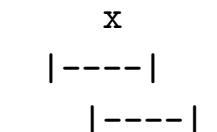
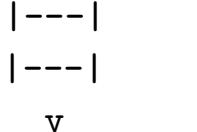
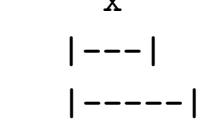
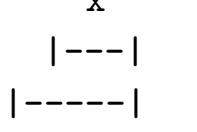
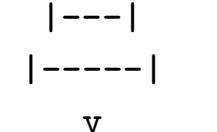
Use intervals only: no points at all.

More cognitively accurate.

Granularity is not fixed.

A “point” is just an interval with nothing inside it.

James Allen's Interval Relations

<code>before(x,y)</code>		<code>meets(x,y)</code>	
	y		y
			x
<code>overlaps(x,y)</code>		<code>equals(x,y)</code>	
	y		y
			x
<code>starts(x,y)</code>		<code>finishes(x,y)</code>	
	y		y
			x
<code>during(x,y)</code>			
	y		

[James F. Allen, Maintaining Knowledge About Temporal Intervals, *Communications of the ACM* 26, 11 (Nov 1983), 832–843.]

A Smaller Set of Temporal Relations

If fewer distinctions are needed, one may use

before(x, y) for Allen's *before*(x, y) \vee *meets*(x, y)

during(x, y) for Allen's *starts*(x, y) \vee *during*(x, y) \vee *finishes*(x, y)

overlaps(x, y) and *equals*(x, y)

and appropriate converses.

7.3 Things *vs.* Substances

Count Nouns *vs.* Mass Nouns

A count noun denotes a thing.

Count nouns can be singular or plural.

Things can be counted.

One dog. Two dogs.

A mass noun denotes a substance.

Mass nouns can only be singular.

One can have a quantity of a substance.

A glass of water. A pint of ice cream.

A Quantity of a Substance is a Thing

water a substance

a lake = a body of water a thing

lakes a plurality of things

40 acres of lakes a quantity of a substance

Nouns with mass and count senses

A noun might have both senses.

a piece of pie vs. *A piece of a pie*

two pieces of steak vs. *two steaks*

Any count noun can be “massified”.

Any thing can be put through “the universal grinder”.

I can't get up; I've got cat on my lap.