Knowledge Representation and Reasoning
Logics for Artificial Intelligence

Stuart C. Shapiro

Department of Computer Science and Engineering
and Center for Cognitive Science
University at Buffalo, The State University of New York
Buffalo, NY 14260-2000

shapiro@cse.buffalo.edu

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Taxonomies: Categories as Intensional Sets

In mathematics, a set is defined by its members.

This is an **extensional set**.

Plato: *Man is a featherless biped.*

An **intensional** set is defined by properties.

Aristotle: *Man is a rational animal.*

A category (type, class) is an intensional set.

Taxonomies: Need for Two Relations

With sets, there's a difference between

set membership, $\in$ $5 \in \{1,3,5,7,9\}$

and subset, $\subset, \subseteq$ $\{1,3,5,7,9\} \subset \{1,2,3,4,5,6,7,8,9\}$

One difference is that subset is transitive:
$\{1,3,5\} \subset \{1,3,5,7,9\}$ and $\{1,3,5,7,9\} \subset \{1,2,3,4,5,6,7,8,9\}$
and $\{1,3,5\} \subset \{1,2,3,4,5,6,7,8,9\}$

membership is not:
$5 \in \{1,3,5,7,9\}$ and $\{1,3,5,7,9\} \in \{\{1,3,5,7,9\},\{2,4,6,8\}\}$
but $5 \notin \{\{1,3,5,7,9\},\{2,4,6,8\}\}$

Similarly, we need both the instance relation and the subcategory relation.

Taxonomies:
Categories as Unary Predicates

One way to represent taxonomies:

$Canary(Tweety)$

$\forall x[(Canary(x) \Rightarrow Bird(x)]$
$\forall x[(Bird(x) \Rightarrow Vertebrate(x)]$
$\forall x[(Vertebrate(x) \Rightarrow Chordate(x)]$
$\forall x[(Chordate(x) \Rightarrow Animal(x)]$

Taxonomies: Reifying

To reify: to make a thing of.

Allows discussion of "predicates" in FOL.

Membership: *Member* or *Instance* or *Isa*

$Isa(Tweety, Canary)$

Subcategory: *Subclass* or *Ako* (sometimes, even, *Isa*)

$Ako(Canary, Bird)$

$Ako(Bird, Vertebrate)$

$Ako(Vertebrate, Chordate)$

$Ako(Chordate, Animal)$

Axioms:

$\forall x \forall c_1 \forall c_2 [Isa(x, c_1) \wedge Ako(c_1, c_2) \Rightarrow Isa(x, c_2)]$

$\forall c_1 \forall c_2 \forall c_3 [Ako(c_1, c_2) \wedge Ako(c_2, c_3) \Rightarrow Ako(c_1, c_3)]$

Discussing Categories

$$Isa(Canary, Species)$$

$$Isa(Bird, Class)$$

$$Isa(Chordate, Phylum)$$

$$Isa(Animal, Kingdom)$$

$$Extinct(Dinosaur)$$

Note: That's *Isa*, not *Ako*.

If categories are predicates, requires second-order logic.

Other relationships: exhaustive subcategories, disjoint categories, partitions.

DAG (directed acyclic graph), rather than just a tree.
E.g., human: man vs. woman; child vs. adult vs. senior.

Transitive Closure

It's sometimes useful (especially in Prolog)
to have a second relation, R_2
be the transitive closure of a relation, R_1.

$$\forall R_1, R_2[transitiveClosureOf(R_2, R_1) \\ \quad \Leftrightarrow [\forall x, y(R_1(x, y) \Rightarrow R_2(x, y)) \\ \quad\quad \wedge \forall x, y, z[R_1(x, y) \wedge R_2(y, z) \Rightarrow R_2(x, z)]]$$

E.g. *ancestor* is the transitive closure of *parent*:

$$\forall x, y[parent(x, y) \Rightarrow ancestor(x, y)]$$

$$\forall x, y, z[parent(x, y) \wedge ancestor(y, z) \Rightarrow ancestor(x, z)]$$

7.2 Time

How would you represent time?

Discuss

Subjective *vs.* Objective: Subjective

Make now an individual in the domain.

Include other times relative to now.

OK if time doesn't move.

Subjective *vs.* Objective: Objective

Make `now` a meta-logical variable with some time-denoting term as value.

Relate times to each other, *e.g.* $Before(t1, t2)$.

Now can move by giving `now` a new value.

Points *vs.* Intervals: Points

Use numbers: integers, rationals, reals?

Computer reals aren't really dense.

How to assign numbers to times?

Granularity: How big, numerically, is a day, or any other interval of time?

If an interval is defined as a pair of points, which interval is the midpoint in, if one interval immediately follows another?

Points *vs.* Intervals: Intervals

Use intervals only: no points at all.

More cognitively accurate.

Granularity is not fixed.

A “point” is just an interval with nothing inside it.

James Allen's Interval Relations

```
               x                              x
before(x,y)  |---|  |---|      meets(x,y)   |---|---|
                      y                           y
                                             x
                   x                       |---|
overlaps(x,y)   |----|         equals(x,y) |---|
                  |----|                     y
                     y
                   x                            x
starts(x,y)      |---|     finishes(x,y)      |---|
                 |-----|                    |-----|
                    y                          y
                  x
during(x,y)     |---|
               |-----|
                  y
```

[James F. Allen, Maintaining Knowledge About Temporal Intervals, *Communications of the ACM 26*, 11 (Nov 1983), 832–843.]

A Smaller Set of Temporal Relations

If fewer distinctions are needed, one may use

$before(x, y)$ for Allen's $before(x, y) \vee meets(x, y)$

$during(x, y)$ for Allen's $starts(x, y) \vee during(x, y) \vee finishes(x, y)$

$overlaps(x, y)$ and $equals(x, y)$

and appropriate converses.

7.3 Things *vs.* Substances Count Nouns *vs.* Mass Nouns

A count noun denotes a thing.
Count nouns can be singular or plural.
Things can be counted.
One dog. Two dogs.

A mass noun denotes a substance.
Mass nouns can only be singular.
One can have a quantity of a substance.
A glass of water. A pint of ice cream.

A Quantity of a Substance is a Thing

water a substance

a lake = a body of water a thing

lakes a plurality of things

40 acres of lakes a quantity of a substance

Nouns with mass and count senses

A noun might have both senses.

a piece of pie vs. *A piece of a pie*

two pieces of steak vs. *two steaks*

Any count noun can be "massified".
Any thing can be put through "the universal grinder".
I can't get up; I've got cat on my lap.