

# Knowledge Representation and Reasoning Logics for Artificial Intelligence

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# 8.1 SNePSLOG Semantics

## The Intensional Domain of (Mental) Entities

Frege: *The Morning Star is the Evening Star.*

different from *The Morning Star is the Morning Star.*

Russell: *George IV wanted to know whether Scott was the author of Waverly.*

not *George IV wanted to know whether the author of Waverly was the author of Waverly.*

Jerry Siegel and Joe Shuster: Clark Kent is a mild-mannered reporter; Superman is the man of steel.

# Intensions vs. Extensions

*the Morning Star and the Evening Star*

*Scott and the author of Waverly*

*Clark Kent and Superman*

are different intensions, or intensional entities, or mental entities, or just entities,

even though they are coreferential, or extensionally equivalent, or have the same extensions.

# SNePSLOG Semantics

## Intensional Representation

SNePSLOG individual ground terms denote intensions, (mental) entities.

Mental entities include propositions.

Propositions are first-class members of the domain.

SNePSLOG wffs denote propositions.

Assume that for every entity in the domain there is a term that denotes it.

Make unique names assumption: no two terms denote the same entity.

# The Knowledge Base

Think of the SNePS KB as the contents of the mind of an intelligent agent.

The terms in the KB denote mental entities that the agent has conceived of (so far).

Some of the wffs are **asserted**.

These denote propositions that the agent believes.

The rules of inference sanction believing some additional proposition(s), but drawing that inference is optional.

I.e., the agent is not logically omniscient.



## 8.2 SNePSLOG Syntax

### Atomic Symbols

Individual Constants, Variables, Function Symbols:

any Lisp symbol, number, or string.

All that matters is the sequence of characters.

I.e. "4", \4, and 4, are the same.

The sets of individual constants, variables, and function symbols should be distinct, but don't have to be.

# SNePSLOG Syntax

## Terms

An individual constant is a term.

A variable is a term.

If  $t_1, \dots, t_n$  are terms, then  $\{t_1, \dots, t_n\}$  is a set of terms.

If  $f$  is a function symbol or a variable, then  $f()$  is a term.

If  $t_1, \dots, t_n$  are terms or sets of terms and  $f$  is a function symbol or variable, then  $f(t_1, \dots, t_n)$  is a term.

A function symbol needn't have a fixed arity, but it might be a mistake of formalization otherwise.

# SNePSLOG Syntax

## Atomic Wffs

If  $x$  is a variable, then  $x$  is a wff.

If  $P$  is a proposition-valued function symbol or variable, then  $P()$  is a wff.

If  $t_1, \dots, t_n$  are terms or sets of terms and  $P$  is a proposition-valued function symbol or variable, then  $P(t_1, \dots, t_n)$  is a wff.

A predicate symbol needn't have a fixed arity, but it might be a mistake of formalization otherwise.

If  $P_1, \dots, P_n$  are wffs, then  $\{P_1, \dots, P_n\}$  is a set of wffs.

Abbreviation: If  $P$  is a wff, then  $P$  is an abbreviation of  $\{P\}$ .

Every wff is a proposition-denoting term.

# SNePSLOG Syntax/Semantics

## AndOr

If  $\{P_1, \dots, P_n\}$  is a set of wffs (proposition-denoting terms), and  $i$  and  $j$  are integers such that  $0 \leq i \leq j \leq n$ , then  $\text{andor}(i, j)\{P_1, \dots, P_n\}$  is a wff (proposition-denoting term).

The proposition that at least  $i$  and at most  $j$  of  $P_1, \dots, P_n$  are True.

# SNePSLOG Syntax/Semantics

## Abbreviations of AndOr

$$\sim P = \text{andor}(0, 0) \{P\}$$

$$\text{and}\{P_1, \dots, P_n\} = \text{andor}(n, n) \{P_1, \dots, P_n\}$$

$$\text{or}\{P_1, \dots, P_n\} = \text{andor}(1, n) \{P_1, \dots, P_n\}$$

$$\text{nand}\{P_1, \dots, P_n\} = \text{andor}(0, n - 1) \{P_1, \dots, P_n\}$$

$$\text{nor}\{P_1, \dots, P_n\} = \text{andor}(0, 0) \{P_1, \dots, P_n\}$$

$$\text{xor}\{P_1, \dots, P_n\} = \text{andor}(1, 1) \{P_1, \dots, P_n\}$$

$$P_1 \text{ and } \dots \text{ and } P_n = \text{andor}(n, n) \{P_1, \dots, P_n\}$$

$$P_1 \text{ or } \dots \text{ or } P_n = \text{andor}(1, n) \{P_1, \dots, P_n\}$$

# SNePSLOG Syntax/Semantics

## Thresh

If  $\{P_1, \dots, P_n\}$  is a set of wffs (proposition-denoting terms) and  $i$  and  $j$  are integers such that  $0 \leq i \leq j \leq n$ , then  $\text{thresh}(i, j) \{P_1, \dots, P_n\}$  is a wff (proposition-denoting term).

The proposition that either fewer than  $i$  or more than  $j$  of  $P_1, \dots, P_n$  are True.

# SNePSLOG Syntax/Semantics

## Abbreviations of Thresh

$\text{iff}\{P_1, \dots, P_n\}$

is an abbreviation of  $\text{thresh}(1, n - 1)\{P_1, \dots, P_n\}$

$P_1 \Leftrightarrow \dots \Leftrightarrow P_n$

is an abbreviation of  $\text{thresh}(1, n - 1)\{P_1, \dots, P_n\}$

$\text{thresh}(i)\{P_1, \dots, P_n\}$

is an abbreviation of  $\text{thresh}(i, n - 1)\{P_1, \dots, P_n\}$

# SNePSLOG Syntax/Semantics

## Numerical Entailment

If  $\{P_1, \dots, P_n\}$  and  $\{Q_1, \dots, Q_m\}$  are sets of wffs (proposition-denoting terms), and  $i$  is an integer,  $1 \leq i \leq n$ , then  $\{P_1, \dots, P_n\} \text{ } i \Rightarrow \{Q_1, \dots, Q_m\}$  is a wff (proposition-denoting term).

The proposition that whenever at least  $i$  of  $P_1, \dots, P_n$  are True, then so is any  $Q_j \in \{Q_1, \dots, Q_m\}$ .



# SNePSLOG Syntax/Semantics

## Abbreviations of Numerical Entailment

$$\{P_1, \dots, P_n\} \Rightarrow \{Q_1, \dots, Q_m\}$$

is an abbreviation of  $\{P_1, \dots, P_n\} \text{ 1} \Rightarrow \{Q_1, \dots, Q_m\}$

$$\{P_1, \dots, P_n\} \text{ v} \Rightarrow \{Q_1, \dots, Q_m\}$$

is also an abbreviation of  $\{P_1, \dots, P_n\} \text{ 1} \Rightarrow \{Q_1, \dots, Q_m\}$

$$\{P_1, \dots, P_n\} \& \Rightarrow \{Q_1, \dots, Q_m\}$$

is an abbreviation of  $\{P_1, \dots, P_n\} \text{ n} \Rightarrow \{Q_1, \dots, Q_m\}$

# SNePSLOG Syntax/Semantics

## Universal Quantifier

If  $P$  is a wff (proposition-denoting term)  
and  $x_1, \dots, x_n$  are variables, then

$\text{all}(x_1, \dots, x_n)(P)$  is a wff (proposition-denoting term).

The proposition that for every sequence of ground terms,  $t_1, \dots, t_n$ ,  
 $P\{t_1/x_1, \dots, t_n/x_n\}$  is True.

# SNePSLOG Syntax/Semantics

## Numerical Quantifier

If  $\mathcal{P}$  and  $\mathcal{Q}$  are sets of wffs,  $x_1, \dots, x_n$  are variables, and  $i, j$ , and  $k$  are integers such that  $0 \leq i \leq j \leq k$ , then

$\text{exists}(i, j, k)(x_1, \dots, x_n)(\mathcal{P} : \mathcal{Q})$  is a wff.

The proposition that there are  $k$  sequences of ground terms,  $t_1, \dots, t_n$ , that satisfy every  $P \in \mathcal{P}$ , and, of them, at least  $i$  and at most  $j$  also satisfy every  $Q \in \mathcal{Q}$ .

# SNePSLOG Syntax/Semantics

## Abbreviations of Numerical Quantifier

$\text{nexists}(-, j, -)(x_1, \dots, x_n)(\mathcal{P}: \mathcal{Q})$

is an abbreviation of  $\text{nexists}(0, j, \infty)(x_1, \dots, x_n)(\mathcal{P}: \mathcal{Q})$

$\text{nexists}(i, -, k)(x_1, \dots, x_n)(\mathcal{P}: \mathcal{Q})$

is an abbreviation of  $\text{nexists}(i, k, k)(x_1, \dots, x_n)(\mathcal{P}: \mathcal{Q})$

# SNePSLOG Syntax/Semantics

## Wffs are Terms

Every wff is a proposition-denoting term.

So, e.g., `Believes(Tom, ~Penguin(Tweety))`  
is a wff, and a well-formed term.

For a more complete, more formal syntax, see  
The *SNePS 2.7.1 User's Manual*,

<http://www.cse.buffalo.edu/sneps/Manuals/manual271.pdf>.

## 8.3 SNePSLOG Proof Theory

### Implemented Rules of Inference

#### Reduction Inference

**Reduction Inference<sub>1</sub>:** If  $\alpha$  is a set of terms and  $\beta \subset \alpha$ ,  
$$P(t_1, \dots, \alpha, \dots t_n) \vdash P(t_1, \dots, \beta, \dots t_n)$$

**Reduction Inference<sub>2</sub>:** If  $\alpha$  is a set of terms, and  $t \in \alpha$ ,  
$$P(t_1, \dots, \alpha, \dots t_n) \vdash P(t_1, \dots, t, \dots t_n)$$

# Example of Reduction Inference

```
: clearkb
```

```
Knowledge Base Cleared
```

```
: Member({Fido, Rover, Lassie}, {dog, pet}).
```

```
  wff1!: Member({Lassie,Rover,Fido},{pet,dog})
```

```
CPU time : 0.00
```

```
: Member ({Fido, Lassie}, dog)?
```

```
  wff2!: Member({Lassie,Fido},dog)
```

```
CPU time : 0.00
```

# SNePSLOG Proof Theory

## Implemented Rules of Inference for AndOr

**AndOr I<sub>1</sub>**:  $P_1, \dots, P_n \vdash \text{andor}(n, n) \{P_1, \dots, P_n\}$

**AndOr I<sub>2</sub>**:  $\sim P_1, \dots, \sim P_n \vdash \text{andor}(0, 0) \{P_1, \dots, P_n\}$

**AndOr E<sub>1</sub>**:  $\text{andor}(i, j) \{P_1, \dots, P_n\}, \sim P_1, \dots, \sim P_{n-i} \vdash P_j$   
for  $n - i < j \leq n$

**AndOr E<sub>2</sub>**:  $\text{andor}(i, j) \{P_1, \dots, P_n\}, P_1, \dots, P_j \vdash \sim P_k,$   
for  $j < k \leq n$



# SNePSLOG Proof Theory

## Implemented Rules of Inference for Thresh

**Thresh E<sub>1</sub>:** When at least  $i$  args are true, and at least  $n - j - 1$  args are false, conclude that any other arg is true.

$$\text{thresh}(i, j) \{P_1, \dots, P_n\},$$

$$P_1, \dots, P_i, \neg P_{i+1}, \dots, \neg P_{i+n-j-1}$$

$$\vdash P_{i+n-j}$$

**Thresh E<sub>2</sub>:** When at least  $i - 1$  args are true, and at least  $n - j$  args are false, conclude that any other arg is false.

$$\text{thresh}(i, j) \{P_1, \dots, P_n\},$$

$$P_1, \dots, P_{i-1}, \neg P_{i+1}, \dots, \neg P_{i+n-j}$$

$$\vdash \neg P_i$$

# SNePSLOG Proof Theory

## Implemented Rules of Inference

### for $\&=>$

$\&=>$ I: If  $\mathcal{A}, P_1, \dots, P_n \vdash Q_i$  for  $1 \leq i \leq m$

then  $\mathcal{A} \vdash \{P_1, \dots, P_n\} \&=> \{Q_1, \dots, Q_m\}$

$\&=>$ E:  $\{P_1, \dots, P_n\} \&=> \{Q_1, \dots, Q_m\}, P_1, \dots, P_n \vdash Q_i,$   
for  $1 \leq i \leq m$

# SNePSLOG Proof Theory

## Implemented Rules of Inference

### for $v \Rightarrow$

$v \Rightarrow$ I: If  $\mathcal{A} \vdash P \ v \Rightarrow Q$  and  $\mathcal{A} \vdash Q \ v \Rightarrow R$  then  $\mathcal{A} \vdash P \ v \Rightarrow R$

$v \Rightarrow$ E:  $\{P_1, \dots, P_n\} \ v \Rightarrow \{Q_1, \dots, Q_m\}$ ,  $P_i \vdash Q_j$ ,  
for  $1 \leq i \leq n, 1 \leq j \leq m$

# SNePSLOG Proof Theory

## Implemented Rules of Inference

for  $i \Rightarrow$

$i \Rightarrow E: \{P_1, \dots, P_n\} i \Rightarrow \{Q_1, \dots, Q_m\}, P_1, \dots, P_i \vdash Q_j,$   
for  $1 \leq i \leq n, 1 \leq j \leq m$

# SNePSLOG Proof Theory

## Implemented Rules of Inference

### for all

Universal Elimination for universally quantified versions of  
andor, thresh,  $v=>$ ,  $&=>$ , and  $i=>$ .

## UVBR & Symmetric Relations

In any substitution  $\{t_1/x_1, \dots, t_n/x_n\}$ , if  $x_i \neq x_j$ , then  $t_i \neq t_j$

: `all(u,v,x,y)(childOf({u,v}, {x,y}) => Siblings({u,v})).`

: `childOf({Tom,Betty,John,Mary}, {Pat,Harry}).`

: `Siblings({?x,?y})?`

wff14!: `Siblings({Mary,John})`

wff13!: `Siblings({John,Betty})`

wff12!: `Siblings({Betty,Tom})`

wff11!: `Siblings({Mary,Betty})`

wff10!: `Siblings({John,Tom})`

wff9!: `Siblings({Mary,Tom})`

# SNePSLOG Proof Theory

## Implemented Rules of Inference

### for nexists

**nexists  $E_1$ :**

$$\begin{array}{l} \text{nexists}(i, j, k)(\bar{x})(\bar{P}(\bar{x}) : Q(\bar{x})), \\ \bar{P}(\bar{t}_1), Q(\bar{t}_1), \dots, \bar{P}(\bar{t}_j), Q(\bar{t}_j), \\ \bar{P}(\bar{t}_{j+1}) \\ \vdash \neg Q(\bar{t}_{j+1}) \end{array}$$

**nexists  $E_2$ :**

$$\begin{array}{l} \text{nexists}(i, j, k)(\bar{x})(\bar{P}(\bar{x}) : Q(\bar{x})), \\ \bar{P}(\bar{t}_1), \neg Q(\bar{t}_1), \dots, \bar{P}(\bar{t}_{k-i}), \neg Q(\bar{t}_{k-i}), \\ \bar{P}(\bar{t}_{k-i+1}) \\ \vdash Q(\bar{t}_{k-i+1}) \end{array}$$

## 8.4 Loading SNePSLOG

```
cl-user(2): :ld /projects/snwiz/bin/sneps
; Loading /projects/snwiz/bin/sneps.lisp
;;; Installing streamc patch, version 2.
Loading system SNePS...10% 20% 30% 40% 50% 60% 70% 80% 90% 100%
SNePS-2.7 [PL:2 2010/09/04 02:35:18] loaded.
Type '(sneps)' or '(snepslog)' to get started.
```

```
cl-user(3): (snepslog)
```

Welcome to SNePSLOG (A logic interface to SNePS)

Copyright (C) 1984--2010 by Research Foundation of  
State University of New York. SNePS comes with ABSOLUTELY NO WARRANTY!  
Type 'copyright' for detailed copyright information.  
Type 'demo' for a list of example applications.



# Running SNePSLOG

cl-user(3): (snepslog)

Welcome to SNePSLOG (A logic interface to SNePS)

Copyright (C) 1984--2010 by Research Foundation of  
State University of New York. SNePS comes with ABSOLUTELY NO WARRANTY!  
Type 'copyright' for detailed copyright information.  
Type 'demo' for a list of example applications.

: clearkb

Knowledge Base Cleared

CPU time : 0.00

: Member({Fido, Rover, Lassie}, {dog, pet}).

wff1!: Member({Lassie,Rover,Fido},{pet,dog})

CPU time : 0.00

: Member ({Fido, Lassie}, dog)?

wff2!: Member({Lassie,Fido},dog)

CPU time : 0.00

# Common SNePSLOG Commands

: clearkb

Knowledge Base Cleared

: all(x)(dog(x) => animal(x)). ; Assert into the KB

wff1!: all(x)(dog(x) => animal(x))

: dog(Fido). ; Assert into the KB

wff2!: dog(Fido)

: dog(Fido)?? ; Query assertion without inference

wff2!: dog(Fido)

# Common SNePSLOG Commands

- : animal(Fido)?? ; Query assertion without inference
  
- : animal(Fido)? ; Query assertion with inference  
wff3!: animal(Fido)
  
- : dog(Rover)! ; Assert into the KB & do forward inference  
wff6!: animal(Rover)  
wff5!: dog(Rover)
  
- : list-asserted-wffs ; Print all asserted wffs  
wff6!: animal(Rover)  
wff5!: dog(Rover)  
wff3!: animal(Fido)  
wff2!: dog(Fido)  
wff1!: all(x)(dog(x) => animal(x))

# Tracing Inference

```
: trace inference
Tracing inference.

: animal(Fido)?

I wonder if wff3:  animal(Fido)
holds within the BS defined by context default-defaultct

I wonder if wff5:  dog(Fido)
holds within the BS defined by context default-defaultct

I know wff2!:  dog({Rover,Fido})

Since wff1!:  all(x)(dog(x) => animal(x))
and wff5!:  dog(Fido)
I infer wff3:  animal(Fido)

    wff3!:  animal(Fido)
CPU time : 0.01

: untrace inference
Untracing inference.
CPU time : 0.00

: animal(Rover)?
wff6!:  animal(Rover)
```

# Recursive Rules

## Don't Cause Infinite Loops

```
: all(x,y)(parentOf(x,y) => ancestorOf(x,y)).
wff1!: all(y,x)(parentOf(x,y) => ancestorOf(x,y))

: all(x,y,z)({ancestorOf(x,y), ancestorOf(y,z)} &=> ancestorOf(x,z)).
wff2!: all(z,y,x)({ancestorOf(y,z), ancestorOf(x,y)} &=> {ancestorOf(x,z)})

: parentOf(Sam,Lou).
wff3!: parentOf(Sam,Lou)

: parentOf(Lou,Stu).
wff4!: parentOf(Lou,Stu)

: ancestorOf(Max,Stu).
wff5!: ancestorOf(Max,Stu)

: ancestorOf(?x,Stu)?
wff8!: ancestorOf(Sam,Stu)
wff6!: ancestorOf(Lou,Stu)
wff5!: ancestorOf(Max,Stu)
CPU time : 0.01
```



# Eager-Beaver Search

```
: all(x)(Duck(motherOf(x)) => Duck(x)).
wff1!: all(x)(Duck(motherOf(x)) => Duck(x))
: all(x)({walksLikeaDuck(x), talksLikeaDuck(x)} &=> Duck(x)).
wff2!: all(x)({talksLikeaDuck(x),walksLikeaDuck(x)} &=> {Duck(x)})
: and{talksLikeaDuck(Daffy),walksLikeaDuck(Daffy)}.
wff5!: walksLikeaDuck(Daffy) and talksLikeaDuck(Daffy)

: Duck(Daffy)? (1)
I wonder if wff6: Duck(Daffy)

I wonder if wff9: Duck(motherOf(Daffy))

I wonder if wff3: talksLikeaDuck(Daffy)

I wonder if wff4: walksLikeaDuck(Daffy)

It is the case that wff4: walksLikeaDuck(Daffy)

It is the case that wff3: talksLikeaDuck(Daffy)

Since wff2!: all(x)({talksLikeaDuck(x),walksLikeaDuck(x)} &=> {Duck(x)})
and wff3!: talksLikeaDuck(Daffy)
and wff4!: walksLikeaDuck(Daffy)
I infer wff6: Duck(Daffy)

wff6!: Duck(Daffy)
CPU time : 0.02
```

# Contradictions

## The KB

: clearkb

Knowledge Base Cleared

: all(x)(nand{Mammal(x), Fish(x)}).

wff1!: all(x)(nand{Fish(x),Mammal(x)})

: all(x)(LivesInWater(x) => Fish(x)).

wff2!: all(x)(LivesInWater(x) => Fish(x))

: all(x)(BearsYoungAlive(x) => Mammal(x)).

wff3!: all(x)(BearsYoungAlive(x) => Mammal(x))

: LivesInWater(whale).

wff4!: LivesInWater(whale)

: BearsYoungAlive(whale).

wff5!: BearsYoungAlive(whale)



# Contradictions

## The Contradiction

: ?x(whale)?

A contradiction was detected within context default-defaulttct.

The contradiction involves the newly derived proposition:

wff6!: Mammal(whale)

and the previously existing proposition:

wff7!: ~Mammal(whale)

You have the following options:

1. [C]ontinue anyway, knowing that a contradiction is derivable;
2. [R]e-start the exact same run in a different context which is not inconsistent;
3. [D]rop the run altogether.

(please type c, r or d)

=><= d

# SNePSLOG Demonstrations

: demo

Available demonstrations:

- 1: Socrates - Is he mortal?
- 2: UVBR - Demonstrating the Unique Variable Binding Rule
- 3: The Jobs Puzzle - A solution with the Numerical Quantifier
- 4: Pegasus - Why winged horses lead to contradictions
- 5: Schubert's Steamroller
- 6: Rule Introduction - Various examples
- 7: Examples of various SNeRE constructs.
- 8: Enter a demo filename

Your choice (q to quit):

## 8.5 Reasoning Heuristics

Logically equivalent SNePSLOG wffs  
are interpreted differently by the SNePS Reasoning System.

## $\vee\Rightarrow$ -Elimination

Instead of

$$\frac{\begin{array}{c} P() \\ (P() \text{ or } Q()) \Rightarrow R() \end{array}}{R()}$$

which would require **or-I** followed by  $\Rightarrow$ -E

Have

$$\frac{\begin{array}{c} P() \\ \{P(), Q()\} \vee\Rightarrow R() \end{array}}{R()}$$

which requires only  $\vee\Rightarrow$ -E

# Example of $v \Rightarrow$ -E

```
: P().  
wff1!: P()  
  
: {P(), Q()} v=> R().  
wff4!: {Q(),P()} v=> {R()}  
  
: trace inference  
Tracing inference.  
  
: R()?  
I wonder if wff3: R()  
holds within the BS defined by context default-defaultct  
  
I wonder if wff2: Q()  
holds within the BS defined by context default-defaultct  
  
I know wff1!: P()  
  
Since wff4!: {Q(),P()} v=> {R()}  
and wff1!: P()  
I infer wff3: R()  
  
wff3!: R()
```

# Bi-Directional Inference

## Backward Inference

```
: {p(), q()} v=> {r(), s()}.  
wff5!: {q(),p()} v=> {s(),r()}
```

```
: p().  
wff1!: p()
```

```
: r()?
```

```
I wonder if wff3: r()  
holds within the BS defined by context default-defaultct
```

```
I wonder if wff2: q()  
holds within the BS defined by context default-defaultct
```

```
I know wff1!: p()
```

```
Since wff5!: {q(),p()} v=> {s(),r()}  
and wff1!: p()  
I infer wff3: r()
```

```
wff3!: r()
```

# Bi-Directional Inference

## Forward Inference

```
: {p(), q()} v=> {r(), s()}.  
wff5!: {q(),p()} v=> {s(),r()}
```

```
: p()!
```

```
Since wff5!: {q(),p()} v=> {s(),r()}  
and wff1!: p()  
I infer wff4: s()
```

```
Since wff5!: {q(),p()} v=> {s(),r()}  
and wff1!: p()  
I infer wff3: r()
```

```
wff4!: s()  
wff3!: r()  
wff1!: p()
```

# Bi-Directional Inference

## Forward-in-Backward Inference

```
: {p(), q()} v=> {r(), s()}.  
wff5!: {q(),p()} v=> {s(),r()}
```

```
: r()?
```

```
I wonder if wff3: r()  
holds within the BS defined by context default-defaultct
```

```
I wonder if wff2: q()  
holds within the BS defined by context default-defaultct
```

```
I wonder if wff1: p()  
holds within the BS defined by context default-defaultct
```

```
: p()!
```

```
Since wff5!: {q(),p()} v=> {s(),r()}  
and wff1!: p()  
I infer wff3: r()
```

```
wff3!: r()  
wff1!: p()
```

Active connection graph cleared by `clear-infer`.



# Bi-Directional Inference

## Backward-in-Forward Inference

```
: p().  
  wff1!: p()  
  
: p() => (q() => r()).  
  wff5!: p() => (q() => r())  
  
: q()!  
  
I know wff1!: p()  
  
Since wff5!: p() => (q() => r())  
and wff1!: p()  
I infer wff4: q() => r()  
  
I know wff2!: q()  
  
Since wff4!: q() => r()  
and wff2!: q()  
I infer wff3: r()  
  
  wff4!: q() => r()  
  wff3!: r()  
  wff2!: q()
```

# Modus Tollens Not Implemented

: all(x) (p(x) => q(x)).  
wff1!: all(x) (p(x) => q(x))

: p(a).  
wff2!: p(a)

: q(a)?  
wff3!: q(a)

: ~q(b).  
wff6!: ~q(b)

: p(b)?  
:

# Use Disjunctive Syllogism Instead

:  $\text{all}(x)(\text{or}\{\sim p(x), q(x)\})$ .

wff1!:  $\text{all}(x)(q(x) \text{ or } \sim p(x))$

:  $p(a)$ .

wff2!:  $p(a)$

:  $q(a)?$

wff3!:  $q(a)$

:  $\sim q(b)$ .

wff7!:  $\sim q(b)$

:  $p(b)?$

wff9!:  $\sim p(b)$

## $\Rightarrow$ Is Not Material Implication

If  $\Rightarrow$  is material implication,

$$\neg(P \Rightarrow Q) \Leftrightarrow (P \wedge \neg Q)$$

and

$$\neg(P \Rightarrow Q) \models P$$

But  $\sim(p \Rightarrow q)$  just means that its not the case that  $p \Rightarrow q$ :

:  $\sim(p() \Rightarrow q())$ .

wff4!:  $\sim(p() \Rightarrow q())$

:  $p()$ ?

:

# Use or Instead of Material Implication

:  $\sim(\sim p() \text{ or } q())$ .

wff5!:  $\text{nor}\{q(), \sim p()\}$

:  $p()?$

wff1!:  $p()$

# Ordering of Nested Rules Matters

## Optimal Order

```
: wifeOf(Caren,Stu).
: wifeOf(Ruth,Mike).
: brotherOf(Stu,Judi).
: brotherOf(Mike,Lou).
: parentOf(Judi,Ken).
: parentOf(Lou,Stu).
: all(w,x)(wifeOf(w,x)
            => all(y)(brotherOf(x,y)
                    => all(z)(parentOf(y,z)
                              => auntOf(w,z))))).
: auntOf(Caren,Ken)?
```

```
I wonder if wff8:  auntOf(Caren,Ken)
I wonder if p7:   wifeOf(Caren,x)
I know wff1!:    wifeOf(Caren,Stu)

I wonder if p8:  brotherOf(Stu,y)
I know wff3!:   brotherOf(Stu,Judi)

I wonder if wff5!: parentOf(Judi,Ken)
I know wff5!:   parentOf(Judi,Ken)

wff8!:  auntOf(Caren,Ken)
CPU time : 0.03
```

# Ordering of Nested Rules Matters

## Bad Order

```
all(x,y)(brotherOf(x,y)
=> all(w)(wifeOf(w,x)
=> all(z)(parentOf(y,z)
=> auntOf(w,z))))).
: auntOf(Caren,Ken)?
```

```
I wonder if wff8:  auntOf(Caren,Ken)
I wonder if p1:  brotherOf(x,y)
I know wff3!:  brotherOf(Stu,Judi)
I know wff4!:  brotherOf(Mike,Lou)
```

```
I wonder if wff12:  wifeOf(Caren,Mike)
I wonder if wff1!:  wifeOf(Caren,Stu)
I know wff1!:  wifeOf(Caren,Stu)
```

```
I wonder if wff5!:  parentOf(Judi,Ken)
I know wff5!:  parentOf(Judi,Ken)
```

```
wff8!:  auntOf(Caren,Ken)
CPU time : 0.04
```

# Ordering of Nested Rules Matters

## Parallel

```
all(w,x,y,z)({wifeOf(w,x),brotherOf(x,y),parentOf(y,z)}  
             &=> auntOf(w,z)).
```

```
: auntOf(Caren,Ken)?
```

```
I wonder if wff8:  auntOf(Caren,Ken)
```

```
I wonder if p5:   parentOf(y,Ken)
```

```
I wonder if p2:   brotherOf(x,y)
```

```
I wonder if p6:   wifeOf(Caren,x)
```

```
I know wff5!:    parentOf(Judi,Ken)
```

```
I know wff3!:    brotherOf(Stu,Judi)
```

```
I know wff4!:    brotherOf(Mike,Lou)
```

```
I know wff1!:    wifeOf(Caren,Stu)
```

```
wff8!:  auntOf(Caren,Ken)
```

```
CPU time : 0.03
```



# Lemmas (Expertise)

## Knowledge Base

```
: all(r)(transitive(r)
      => all(x,y,z)({r(x,y),r(y,z)} &=> r(x,z))).
: transitive(biggerThan).
: biggerThan(elephant,lion).
: biggerThan(lion,hyena).
: biggerThan(hyena,rat).
```

# Lemmas: First Task

```
: biggerThan(?x, rat)?  
I wonder if p6: biggerThan(x, rat)  
I know wff5!: biggerThan(hyena, rat)  
I wonder if wff2!: transitive(biggerThan)  
I know wff2!: transitive(biggerThan)  
I infer wff6: all(z, y, x)({biggerThan(x, y), biggerThan(y, z)} => {biggerThan(x, z)})  
I wonder if p8: biggerThan(y, rat)  
I wonder if p10: biggerThan(x, y)  
I know wff5!: biggerThan(hyena, rat)  
I wonder if p12: biggerThan(rat, z)  
I know wff3!: biggerThan(elephant, lion)  
I know wff4!: biggerThan(lion, hyena)  
I infer wff7: biggerThan(lion, rat)  
I infer wff8: biggerThan(elephant, rat)  
...  
wff8!: biggerThan(elephant, rat)  
wff7!: biggerThan(lion, rat)  
wff5!: biggerThan(hyena, rat)  
CPU time : 0.09
```

# Second Task

```
: clear-infer
: biggerThan(truck,SUV).
: biggerThan(SUV,sedan).
: biggerThan(sedan,roadster).

: biggerThan(?x,roadster)?
I wonder if p14: biggerThan(x,roadster)
I know wff11!: biggerThan(sedan,roadster)
I wonder if p10: biggerThan(x,y)
I wonder if p16: biggerThan(y,roadster)
I know wff3!: biggerThan(elephant,lion)
I know wff4!: biggerThan(lion,hyena)
I know wff5!: biggerThan(hyena,rat)
I know wff7!: biggerThan(lion,rat)
I know wff8!: biggerThan(elephant,rat)
I know wff9!: biggerThan(truck,SUV)
I know wff10!: biggerThan(SUV,sedan)
I know wff11!: biggerThan(sedan,roadster)
I infer wff12: biggerThan(SUV,roadster)
I infer wff13: biggerThan(truck,roadster)
I wonder if p17: biggerThan(roadster,z)
  wff13!: biggerThan(truck,roadster)
  wff12!: biggerThan(SUV,roadster)
  wff11!: biggerThan(sedan,roadster)
CPU time : 0.04
```

# Contexts

```
: demo /projects/shapiro/CSE563/Examples/SNePSLOG/facultyMeeting.snepslog
...
: ;;; Example of Contexts
;;; from
;;; J. P. Martins & S. C. Shapiro, Reasoning in Multiple Belief Spaces IJCAI-83, 370-373.

: all(x)(meeting(x) => xor{time(x,morning), time(x,afternoon)}).
  wff1!: all(x)(meeting(x) => (xor{time(x,afternoon),time(x,morning)}))
: all(x,y)({meeting(x),meeting(y)} &=> all(t)(xor{time(x,t),time(y,t)})).
  wff2!: all(y,x)({meeting(y),meeting(x)} &=> {all(t)(xor{time(y,t),time(x,t)}))}
: meeting(facultyMeeting).
  wff3!: meeting(facultyMeeting)
: meeting(seminar).
  wff4!: meeting(seminar)
: meeting(tennisGame).
  wff5!: meeting(tennisGame)
: time(seminar,morning).
  wff6!: time(seminar,morning)
: time(tennisGame,afternoon).
  wff7!: time(tennisGame,afternoon)
: set-context stuSchedule {wff1,wff2,wff3,wff4,wff6}
((assertions (wff6 wff4 wff3 wff2 wff1)) (named (stuSchedule)) (kinconsistent nil))
: set-context tonySchedule {wff1,wff2,wff3,wff5,wff7}
((assertions (wff7 wff5 wff3 wff2 wff1)) (named (tonySchedule)) (kinconsistent nil))
: set-context patSchedule {wff1,wff2,wff3,wff4,wff5,wff6,wff7}
((assertions (wff7 wff6 wff5 wff4 wff3 wff2 wff1)) (named (patSchedule default-defaulttct)) (kinconsistent nil))
```

# Stu's Schedule

```
: set-default-context stuSchedule
((assertions (wff6 wff4 wff3 wff2 wff1)) (named (stuSchedule))
 (kinconsistent nil))

: list-asserted-wffs
wff6!:  time(seminar,morning)
wff4!:  meeting(seminar)
wff3!:  meeting(facultyMeeting)
wff2!:  all(y,x)({meeting(y),meeting(x)}
                &=> {all(t)(xor{time(y,t),time(x,t)}}))
wff1!:  all(x)(meeting(x)
              => (xor{time(x,afternoon),time(x,morning)}))

: time(facultyMeeting,?t)?
wff10!: time(facultyMeeting,afternoon)
wff9!:  ~time(facultyMeeting,morning)
```

# Tony's Schedule

```
: set-default-context tonySchedule
((assertions (wff7 wff5 wff3 wff2 wff1)) (named (tonySchedule))
 (kinconsistent nil))

: list-asserted-wffs
wff12!: xor{time(facultyMeeting,afternoon),time(facultyMeeting,morning)}
wff7!: time(tennisGame,afternoon)
wff5!: meeting(tennisGame)
wff3!: meeting(facultyMeeting)
wff2!: all(y,x)({meeting(y),meeting(x)}
                &=> {all(t)(xor{time(y,t),time(x,t)}}))
wff1!: all(x)(meeting(x)
                => (xor{time(x,afternoon),time(x,morning)}))

: time(facultyMeeting,?t)?
wff11!: ~time(facultyMeeting,afternoon)
wff8!: time(facultyMeeting,morning)
```

# Pat's Schedule

```
: set-default-context patSchedule
((assertions (wff7 wff6 wff5 wff4 wff3 wff2 wff1))
 (named (patSchedule default-defaultct) (kinconsistent nil)))
```

```
: time(facultyMeeting,?t)?
```

A contradiction was detected within context patSchedule.  
The contradiction involves the newly derived proposition:  
wff8!: time(facultyMeeting,morning)

and the previously existing proposition:  
wff9!: ~time(facultyMeeting,morning)

You have the following options:

1. [C]ontinue anyway, knowing that a contradiction is derivable;
2. [R]e-start the exact same run in a different context which is not inconsistent;
3. [D]rop the run altogether.

(please type c, r or d)

=><= d

# Resulting Contexts

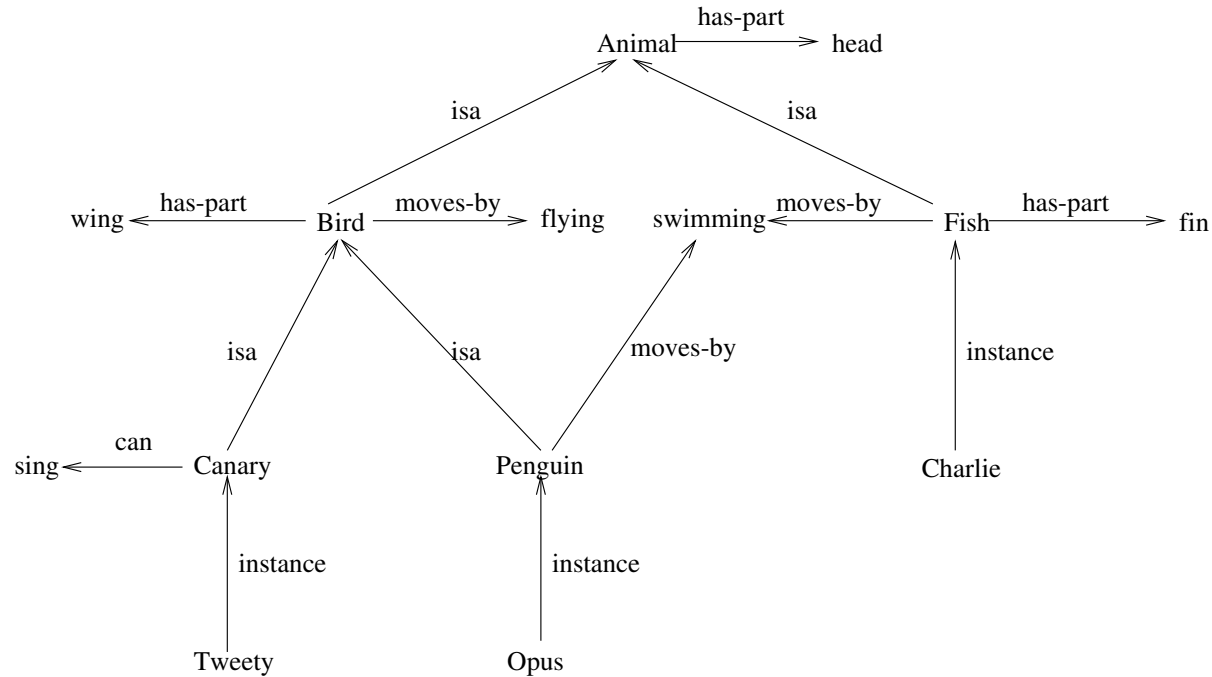
```
: describe-context stuSchedule  
((assertions (wff6 wff4 wff3 wff2 wff1)) (named (stuSchedule))  
 (kinconsistent nil))
```

```
: describe-context tonySchedule  
((assertions (wff7 wff5 wff3 wff2 wff1)) (named (tonySchedule))  
 (kinconsistent nil))
```

```
: describe-context patSchedule  
((assertions (wff7 wff6 wff5 wff4 wff3 wff2 wff1))  
 (named (patSchedule default-defaulttct)) (kinconsistent t))
```



## 8.6 SNePS as a Network: Semantic Networks



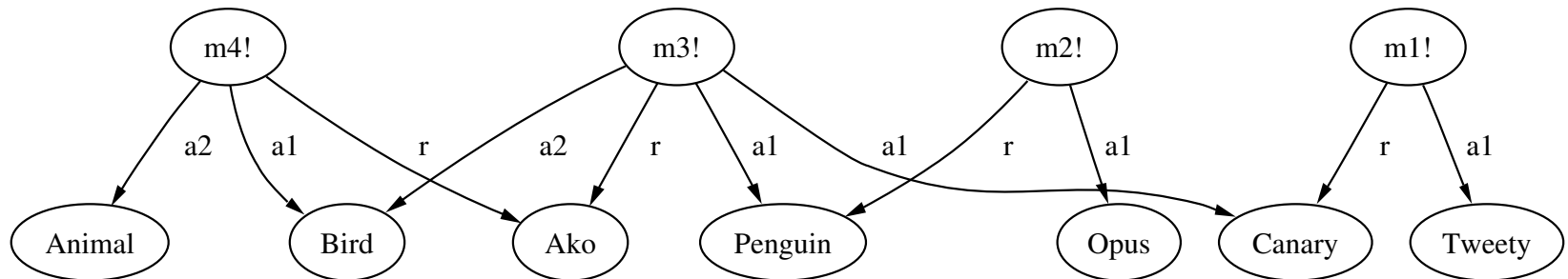
Some psychological evidence.

More efficient search than logical inference.

Unclear semantics.

# SNePS as a Network

```
: clearkb  
: Canary(Tweety).  
: Penguin(Opus).  
: Ako({Canary,Penguin}, Bird).  
: Ako(Bird, Animal).  
: show
```



# Defining Case Frames

```
: set-mode-3
```

```
Net reset
```

```
In SNePSLOG Mode 3.
```

```
Use define-frame <pred> <list-of-arc-labels>.
```

```
...
```

```
: define-frame Canary(class member) "[member] is a [class]"
```

```
Canary(x1) will be represented by {<class, Canary>, <member, x1>}
```

```
: define-frame Penguin(class member) "[member] is a [class]"
```

```
Penguin(x1) will be represented by {<class, Penguin>, <member, x1>}
```

```
: define-frame Ako(nil subclass superclass) "Every [subclass] is a [superclas
```

```
Ako(x1, x2) will be represented by {<subclass, x1>, <superclass, x2>}
```

# Entering the KB

: Canary(Tweety).

wff1!: Canary(Tweety)

: Penguin(Opus).

wff2!: Penguin(Opus)

: Ako({Canary, Penguin}, Bird).

wff3!: Ako({Penguin, Canary}, Bird)

: Ako(Bird, Animal).

wff4!: Ako(Bird, Animal)

# The Knowledge Base

: list-terms

wff1!: Canary(Tweety)

wff2!: Penguin(Opus)

wff3!: Ako({Penguin, Canary}, Bird)

wff4!: Ako(Bird, Animal)

: describe-terms

Tweety is a Canary.

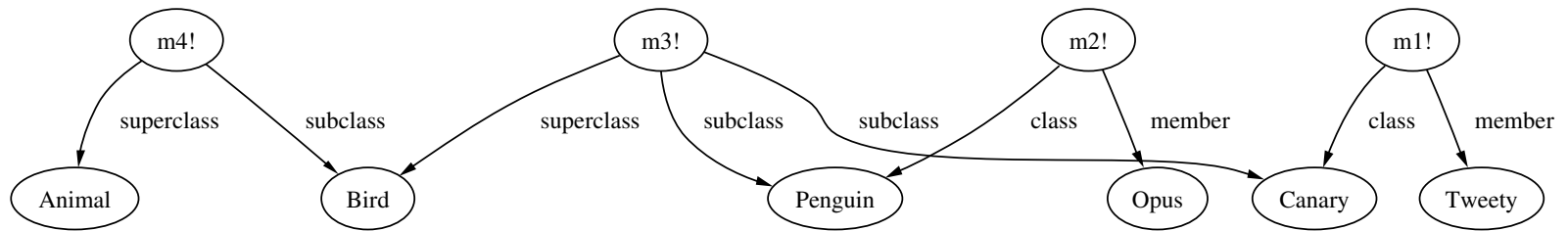
Opus is a Penguin.

Every Penguin and Canary is a Bird.

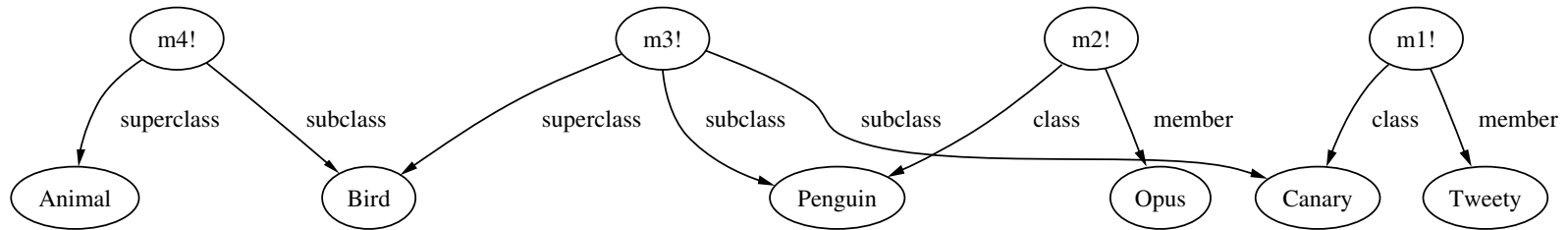
Every Bird is a Animal.

# The Network

: show



# Path-Based Inference



```

: define-path class (compose class
                    (kstar (compose subclass- ! superclass)))
class implied by the path (compose class
                          (kstar
                           (compose subclass- ! superclass)))
class- implied by the path (compose
                          (kstar (compose superclass-
                                       ! subclass))
                          class-)
  
```

# Using Path-Based Inference

```
: list-asserted-wffs
wff4!: Ako(Bird,Animal)
wff3!: Ako({Penguin,Canary},Bird)
wff2!: Penguin(Opus)
wff1!: Canary(Tweety)

: define-frame Animal(class member) "[member] is a [class]"
Animal(x1) will be represented by {<class, Animal>, <member, x1>}

: trace inference
Tracing inference.

: Animal(Tweety)?
I wonder if wff5: Animal(Tweety)
holds within the BS defined by context default-defaultct
I know wff1!: Canary(Tweety)
wff5!: Animal(Tweety)
```



# Rules About Functions in Mode 3

```
: set-mode-3
: define-frame WestOf(relation domain range)
: define-frame isAbove(relation domain range)
: define-frame Likes(relation liker likee)

: define-frame r(relation domain range)
: define-frame anti-symmetric(nil antisymm)

: all(r)(anti-symmetric(r) => all(x,y)(r(x,y) => ~r(y,x))).
wff1!: all(r)(anti-symmetric(r) => (all(y,x)(r(x,y) => (~r(y,x)))))

: anti-symmetric({WestOf, isAbove, Likes}).

: WestOf(Buffalo,Rochester).
: isAbove(penthouse37,lobby37).
: Likes(Betty,Tom).

: WestOf(?x,?y)?
wff9!: ~WestOf(Rochester,Buffalo)
wff3!: WestOf(Buffalo,Rochester)

: isAbove(?x,?y)?
wff13!: ~isAbove(lobby37,penthouse37)
wff4!: isAbove(penthouse37,lobby37)

: Likes(?x,?y)?
wff5!: Likes(Betty,Tom)
```

# Procedural Attachment in SNePS

```
cl-user(3): (snepslog)
: load /projects/snwiz/Libraries/expressions.snepslog

: define-frame Value(nil obj val) "the value of [obj] is [val]"

: define-frame radius(nil radiusof) "the radius of [radiusof]"

: define-frame volume(nil volumeof) "the volume of [volumeof]"

: all(x,r,p)({Value(radius(x), r), Value(pi,p)}
             &=> all(v)(is(v,/(*(4.0,* (p,* (r,* (r,r))))),3.0))
             => Value(volume(x),v))).

: Value(pi,3.14159).

: Value(radius(sphere1), 9.0).

: Value(volume(sphere1), ?x)?
wff13!: Value(volume(sphere1),3053.6257)
```

## 8.7 SNeRE: The SNePS Rational Engine

### Motivation

Coming to believe something  
is different from acting.

# Prolog Searches In Order

## The KB

```
| ?- [user].  
% consulting user...  
| q(X) :- q1(X), q2(X).  
| q1(X) :- p(X), s(X).  
| q2(X) :- r(X), s(X).  
| s(X) :- t(X).  
| p(a).  
| r(a).  
| t(a).  
|  
% consulted user in module user, 0 msec 1592 bytes  
yes
```

# Prolog Searches In Order

## The Run

```
| ?- trace.  
% The debugger will first creep -- showing everything (trace)  
yes  
% trace  
| ?- q(a).  
    1      1 Call: q(a) ?  
    2      2 Call: q1(a) ?  
    3      3 Call: p(a) ?  
    3      3 Exit: p(a) ?  
    4      3 Call: s(a) ?  
    5      4 Call: t(a) ?  
    5      4 Exit: t(a) ?  
    4      3 Exit: s(a) ?  
    2      2 Exit: q1(a) ?  
    6      2 Call: q2(a) ?  
    7      3 Call: r(a) ?  
    7      3 Exit: r(a) ?  
    8      3 Call: s(a) ?  
    9      4 Call: t(a) ?  
    9      4 Exit: t(a) ?  
    8      3 Exit: s(a) ?  
    6      2 Exit: q2(a) ?  
    1      1 Exit: q(a) ?  
  
yes
```

# SNePS Avoids Extra Search

## The KB

: clearkb

Knowledge Base Cleared

: all(x)({q1(x), q2(x)} &=> q(x)).

: all(x)({p(x), s(x)} &=> q1(x)).

: all(x)({r(x), s(x)} &=> q2(x)).

: all(x)(t(x) => s(x)).

: p(a).

: r(a).

: t(a).

# SNePS Avoids Extra Search

## The Search

: trace inference  
Tracing inference.

: q(a)?  
I wonder if wff8: q(a)  
I wonder if wff10: q2(a)  
I wonder if wff12: q1(a)  
I wonder if wff14: s(a)  
I wonder if wff6!: r(a)  
I wonder if wff14: s(a)  
I wonder if wff5!: p(a)  
I know wff6!: r(a)  
I know wff5!: p(a)  
I wonder if wff7!: t(a)  
I know wff7!: t(a)

# SNePS Avoids Extra Search

## The Answers

```
Since wff4!: all(x)(t(x) => s(x))
and wff7!: t(a)
I infer wff14: s(a)
```

```
Since wff3!: all(x)({s(x),r(x)} &=> {q2(x)})
and wff14!: s(a)
and wff6!: r(a)
I infer wff10: q2(a)
```

```
Since wff2!: all(x)({s(x),p(x)} &=> {q1(x)})
and wff14!: s(a)
and wff5!: p(a)
I infer wff12: q1(a)
```

```
Since wff1!: all(x)({q2(x),q1(x)} &=> {q(x)})
and wff10!: q2(a)
and wff12!: q1(a)
I infer wff8: q(a)
```

```
wff8!: q(a)
```



# Primitive Acts

```
: set-mode-3
Net reset
In SNePSLOG Mode 3.
Use define-frame <pred> <list-of-arc-labels>.
...

: define-frame say(action line)
say(x1) will be represented by {<action, say>, <line, x1>}

: ^^
--> (define-primaction sayaction ((line))
      (format sneps:outunit "~A" line))
sayaction

--> (attach-primaction say sayaction)
t

--> ^^

: perform say("Hello world")
Hello world
```

# Effects: The KB

```
: set-mode-3
Net reset
In SNePSLOG Mode 3.
Use define-frame <pred> <list-of-arc-labels>.
...
Effect(x1, x2) will be represented by {<act, x1>, <effect, x2>}
...

: define-frame say (action line)
: define-frame said (act agent object)
: define-frame Utterance (class member)
: ^^
--> (define-primaction sayaction ((line))
      (format sneps:outunit "~A" line))
sayaction
-->
(attach-primaction say sayaction)
t
--> ^^
: Utterance("Hello world").
: all(x)(Utterance(x) => Effect(say(x), said(I,x))).
```

# Effects: The Run

```
: list-asserted-wffs
```

```
wff2!: all(x)(Utterance(x) => Effect(say(x),said(I,x)))
```

```
wff1!: Utterance>Hello world)
```

```
: perform say("Hello world")
```

```
Hello world
```

```
: list-asserted-wffs
```

```
wff5!: Effect(say>Hello world),said(I>Hello world))
```

```
wff4!: said(I>Hello world)
```

```
wff2!: all(x)(Utterance(x) => Effect(say(x),said(I,x)))
```

```
wff1!: Utterance>Hello world)
```

# Defined Acts

```
: set-mode-3
...
ActPlan(x1, x2) will be represented by {<act, x1>, <plan, x2>}
...

: define-frame say (action part1 part2)
: define-frame greet (action object)
: define-frame Person (class member)

: ^^
--> (define-primaction sayaction ((part1) (part2))
      (format sneps:outunit "~A ~A%"
              part1 part2))
sayaction
-->
(attach-primaction say sayaction)
t
--> ^^

: all(x)(Person(x) => ActPlan(greet(x), say>Hello,x))).
: Person(Mike).

: perform greet(Mike).
Hello Mike
```

# Other Propositions about Acts

GoalPlan( $p$ ,  $a$ )

Precondition( $a$ ,  $p$ )

# Control Acts

```
achieve(p)  
do-all({a1, ..., an})  
do-one({a1, ..., an})  
snif({if(p1, a1), ..., if(pn, an)[, else(da)]})  
sniterate({if(p1, a1), ..., if(pn, an)[, else(da)]})  
snsequence(a1, a2)  
withall(x, p(x), a(x) [, da])  
withsome(x, p(x), a(x) [, da])
```

Must use `attach-primaction` on whichever you want to use.

# Policies

`ifdo( $p$ ,  $a$ )`

`whendo( $p$ ,  $a$ )`

`wheneverdo( $p$ ,  $a$ )`

# Mental Acts

believe( $p$ )

disbelieve( $p$ )

adopt( $p$ )

unadopt( $p$ )



# The Execution Cycle

```
perform(act):
  pre := {p | Precondition(act,p)};
  notyet := pre - {p | p ∈ pre & ⊢ p};
  if notyet ≠ nil
    then perform(snsequence(do-all({a | p ∈ notyet
                                   & a = achieve(p)}),
                           act))
  else {effects := {p | Effect(act,p)};
       if act is primitive
         then apply(primitive-function(act), objects(act));
         else perform(do-one({p | ActPlan(act,p)}))
       believe(effects)}
```

# Examples

SNePSLOG demo #7

/projects/robot/Karel/ElevatorWorld/elevator.snepslog

/projects/robot/Karel/DeliveryWorld/DeliveryAgent.snepslog

/projects/robot/Karel/WumpusWorld/WWAgent.snepslog

/projects/robot/Fevahr/Ascii/afevahr.snepslog

/projects/robot/Fevahr/Java/jfevahr.snepslog

/projects/robot/Greenfoot/ElevatorWorld/sneps/elevator.snepslog

/projects/robot/Greenfoot/WumpusWorld/sneps/WWAgent.snepslog