

Proofs of $A \Rightarrow A$

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1 Hilbert-Style¹

Axioms

- A1** $(A \Rightarrow (B \Rightarrow A))$
A2 $((A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \Rightarrow B) \Rightarrow (A \Rightarrow C)))$
A3 $((\neg B \Rightarrow \neg A) \Rightarrow ((\neg B \Rightarrow A) \Rightarrow B))$

Rule of Inference (Modus Ponens) $\frac{A, A \Rightarrow B}{B}$

Proof of $A \Rightarrow A$

- | | | |
|----|---|---------|
| 1. | $(A \Rightarrow ((A \Rightarrow A) \Rightarrow A)) \Rightarrow ((A \Rightarrow (A \Rightarrow A)) \Rightarrow (A \Rightarrow A))$ | A2 |
| 2. | $(A \Rightarrow ((A \Rightarrow A) \Rightarrow A))$ | A1 |
| 3. | $(A \Rightarrow (A \Rightarrow A)) \Rightarrow (A \Rightarrow A)$ | MP, 1,2 |
| 4. | $(A \Rightarrow (A \Rightarrow A))$ | A1 |
| 5. | $A \Rightarrow A$ | MP, 3,4 |

2 Fitch-Style Natural Deduction Proof

1.		A	Hyp
2.		A	Reit, 1
3.		$A \Rightarrow A$	\Rightarrow I, 1-2

¹From Elliott Mendelson, *Introduction to Mathematical Logic*, D. Van Nostrand, Princeton, 1964, 31–32.