CSE 431/531 Homework 4

Your Name:_____ Your University ID:_____

Problems	1	2	3	Total
Max. Score	10	15	15	40
Your Score				

Problem 1 (10 points). Prove that if NP $\neq$ CO-NP, then NP$\backslash$CO-NP $\neq \emptyset$ and CO-NP$\backslash$NP $\neq \emptyset$.

Problem 2 (15 points). For each of the following problem X, answer: whether (1) $X \in$ NP, (2) $X \in$ CO-NP. Each answer is either "yes" or "we do not know". If your answer is yes, you need to give the certifier and the certificate for the proof.

(a) Given a graph $G = (V, E)$ and a positive integer $s \leq |V|$, whether the size of the maximum independent set of G is *at least* s.

(b) Given a graph $G = (V, E)$ and a positive integer $s \leq |V|$, whether the size of the maximum independent set of G is *at most* s.

(c) Given a graph $G = (V, E)$ and a positive integer $s \leq |V|$, whether the size of the maximum independent set of G is *exactly* s.

(d) Given two circuits C_1 and C_2, each with m input variables $z_1, z_2, \cdots, z_m$, decide if the two circuits compute the same function. That is, whether C_1 and C_2 give the same output for every boolean assignment of z-variables.

(e) Given two $\{0,1\}$-strings A and B of length n, and an integer $L \geq 0$, whether the length of the longest common subsequence of A and B has length exactly L.

Problem 3 (15 points). Given a graph $G = (V, E)$, the degree-3 spanning tree (D3ST) problem asks whether G contains a spanning tree T (recall that a spanning tree is a tree in G that contains all the vertices V) of degree at most 3. (The degree of a vertex v in a spanning tree T is the number of edges incident to v in T; the degree of T is the maximum degree of v, over all verticies $v \in V$.) Prove Hamiltonian-Path $\leq_P$ D3ST.