CSE 431/531 Homework 5

Your Name:________________ Your University ID:________________

Problems	1	2	3	Total
Max. Score	12	12	16	40
Your Score				

Problem 1 (12 points). In the max-cut problem, we are given a graph $G = (V, E)$ and we need to output a set $S \subseteq V$ of vertices, such that the number of edges between S and $V \setminus S$ is maximized. That is, we want to maximize $|\{(u, v) \in E : u \in S, v \notin S \text{ or } v \in S, u \notin S\}|$. Design and analyze a 1/2-approximation algorithm for the max-cut problem.

Problem 2 (12 points). In the class, we analyzed a variant of the randomized quicksort problem (where in the "divide" step, we repeatedly choose a random pivot until it breaks the array into two arrays of size at most $3n/4$). Modify the algorithm to give a randomized algorithm for the selection problem that runs in expected $O(n)$ time. Recall that in the selection problem, we are given an array A of n numbers and an index i and we need to output the i-th smallest number in A.

Problem 3 (16 points). Recall that in the set-cover problem, we are given a family of m subsets $S_1, S_2, \cdots, S_m$ of $\{1, 2, \cdots, n\}$. The goal of the problem is to find a minimum-size sub-family of the subsets whose union is $\{1, 2, \cdots, n\}$. That is, find a minimum-size set $I \subseteq \{1, 2, 3, \cdots, m\}$ such that $\bigcup_{i \in I} S_i = \{1, 2, \cdots, n\}$. Assume that every element $j \in \{1, 2, \cdots, n\}$ appears in at most f of the m subsets. Design and analyze an f-approximation algorithm for this problem.