

**Homework 3***Instructor: Shi Li***Deadline: 3/30/2018**

Problems	1	2	3	Total Score
Max. Score	15	15	40	70
Your Score				

**Collaboration Policy** You are allowed to discuss the homework problems with classmates. However, it is highly recommended that you first think about each problem for enough time before the discussion. You must write your solutions by yourself, in your own words. You need to write down the names of the students you collaborated with. For the programming problem, you must implement the algorithm by yourself.

**Problem 1 (15 points)** This is Exercise 2 from the textbook. We consider the following problem of counting significant inversions. Given an array  $A$  of  $n$  positive integers, a pair  $i, j \in \{1, 2, 3, \dots, n\}$  of indices is called a significant inversion if  $i < j$  and  $A[i] > 2A[j]$ . The goal of the problem is to count the number of significant inversions for a given array  $A$ . Give a divide-and-conquer algorithm that runs in  $O(n \lg n)$  time to solve the problem.

**Problem 2 (15 points)** This problem asks you to find the largest rectangle in a histogram, given to you as an array of  $n$  non-negative numbers. For example, if the input is  $(3, 5, 10, 11, 20, 4, 8, 10)$ , then the largest rectangle has size 30 (with height 10 and width 3, covering column 3 to column 5.) Design an  $O(n \lg n)$ -time divide-and-conquer algorithm to solve the problem.

**Problem 3 (40 points)** This is a programming problem. Consider a sequence of numbers defined using the following recursion:

$$F_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ 2 & \text{if } n = 2 \\ F_{n-3} + 2F_{n-2} + F_{n-1} & \text{if } n \geq 3 \end{cases}$$

For example, the first few numbers in the sequence is  $0, 1, 2, 4, 9, 19, 41, 88, \dots$ . Given an integer  $n$  ( $0 \leq n \leq 10^{16}$ ), you need to output  $F_n \bmod 10007$ . Read the number  $n$  from the console and you need to output  $F_n \bmod 10007$  to the console.

Here are some sample inputs and outputs:

<b>Input</b>	4	1000	10000000000
<b>Output</b>	9	7208	9383