CSE 431/531: Algorithm Analysis and Design Spring 2018

Homework 5

Instructor: Shi Li **Deadline: 5/7/2018**

Problems	1	2	3	Total Score
Max. Score	15	30	25	70
Your Score				

Collaboration Policy You are allowed to discuss the homework problems with classmates. However, it is highly recommended that you first think about each problem for enough time before the discussion. You must write your solutions by yourself, in your own words. You need to write down the names of the students you collaborated with.

Problem 1 (15 points) When defining problems in NP, we only consider decision problems. In this problem you need to show that finding a Hamiltonian cycle is not much harder than deciding if a graph contains a Hamiltonian cycle or not.

Formally, suppose you are given a polynomial-time black-box algorithm that decides whether a given graph H contains a Hamiltonian cycle or not. Design a polynomial time algorithm that, given a graph G which is promised to contain a Hamiltonian cycle, outputs a Hamiltonian cycle of G.

Problem 2 (30 points) For each of the following problems, state (1) whether the problem is known to be in NP, and (2) whether the problem is known to be in Co-NP. If your answer is yes, you should briefly describe the certificate and the efficient certifier.

(a) Given a graph $G = (V, E)$ and $s \leq |V|$, the problem asks whether the size of the maximum independent set is *at most* s.

(b) Given a graph $G = (V, E)$ and $s \leq |V|$, the problem asks whether the size of the maximum independent set is *exactly* s.

(c) Given two circuits C_1 and C_2, each with m input variables $z_1, z_2, \cdots, z_m$, decide if the two circuits compute the same function. That is, whether C_1 and C_2 give the same output for every boolean assignment of z-variables.

(d) Given a graph $G = (V, E)$, decide if G is 3-colorable.

(e) Given a graph $G = (V, E)$, decide if G is 2-colorable.

(f) An undirected graph $G = (V, E)$ is called a 1-expander if for every $S \subseteq V$, the number of edges between S and $V \setminus S$ is at least $\min\{|S|, |V \setminus S|\}$. Given a graph G, decide if G is a 1-expander.

Problem 3 (25 points) In class, we proved that HP (Hamiltonian Path) $\leq_P$ HC (Hamiltonian Cycle). Prove the other direction, i.e, HC $\leq_P$ HP.