

**Homework 6***Instructor: Shi Li***Deadline: 5/15/2018**

Problems	1	2	Total Score
Max. Score	25	25	50
Your Score			

**Collaboration Policy** You are allowed to discuss the homework problems with classmates. However, it is highly recommended that you first think about each problem for enough time before the discussion. You must write your solutions by yourself, in your own words. You need to write down the names of the students you collaborated with.

**Problem 1 (25 points)** In the weighted set-cover problem, we are given a family of  $m$  subsets  $S_1, S_2, \dots, S_m$  of  $\{1, 2, \dots, n\}$ , where each set  $S_i$  has a weight  $w_i \geq 0$ . The goal of the problem is to find minimum-weight sub-family of the subsets whose union is  $\{1, 2, \dots, n\}$ . That is, find a set  $I \subseteq \{1, 2, 3, \dots, m\}$  such that  $\bigcup_{i \in I} S_i = \{1, 2, \dots, n\}$  so as to minimize  $\sum_{i \in I} w_i$ . Assume that every element  $j \in \{1, 2, \dots, n\}$  appears in at most  $f$  of the  $m$  subsets. Design and analyze an  $f$ -approximation algorithm for this problem.

**Problem 2 (25 points)** Give the optimal solution for the following linear programming and prove that the solution is optimal. You can use an online LP solver to solve the LP; however your proof of optimality should not depend on the LP solver (i.e, you can not simply say that the LP solver is correct and it gives this solution.)

$$\begin{aligned}
 &\text{minimize } 3x_1 + 2x_2 + 5x_3 \\
 &2x_1 + 2x_2 - 3x_3 \geq 5 \\
 &x_1 - x_2 + 2x_3 \geq 10 \\
 &3x_1 + 5x_2 + x_3 \geq 8 \\
 &x_1, x_2, x_3 \geq 0
 \end{aligned}$$