CSE 431/531: Algorithm Analysis and Design Spring 2019

Homework 3

Instructor: Shi Li **Deadline: 4/11/2019**

Your Name: _____ Your Student ID: _____

Problems	1	2	3	4	Total
Max. Score	10	15	15	40	80
Your Score					

Problem 1 (10 points) For each of the following recurrences, using the master theorem to give the asymptotically tight upper bound.

(a) $T(n) = 4T(n/4) + O(n)$.

(b) $T(n) = 3T(n/3) + O(n)$.

(c) $T(n) = 4T(n/2) + O(n^3\sqrt{n})$.

(d) $T(n) = 5T(n/2) + O(n)$.

Problem 2 (15 points) Consider a sequence of numbers defined using the following recursion:

$$F_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ 2 & \text{if } n = 2 \\ F_{n-3} + 2F_{n-2} + F_{n-1} & \text{if } n \geq 3 \end{cases}.$$

The first few numbers in the sequence is $0, 1, 2, 4, 9, 19, 41, 88, \cdots$. Given an integer n, you need to output F_n. Assume you are given the implementation of the BigInteger class; each object of the class holds an integer as large as F_n; the basic operations such addition, subtraction and multiplication for BigInteger class are also provided to you. Design an algorithm to compute F_n that uses $O(\log n)$ basic operations over the BigInteger class.

Problem 3(15 points) Given two sorted arrays A and B with total size n, you need to design and analyze an $O(\log n)$-time algorithm that outputs the median of the n numbers in A and B. You can assume n is odd and all the numbers are distinct. For example, if $A = [3, 5, 12, 18, 50]$ and $B = [2, 7, 11, 30]$, then you need to output 11 since the set of numbers are $[2, 3, 5, 7, 11, 12, 18, 30, 50]$.

Problem 4 (40 points) We consider the following problem of counting strong inversions. Given an array A of n positive integers, a pair $i, j \in \{1, 2, 3, \cdots, n\}$ of indices is called a strong inversion if $i < j$ and $A[i] > 2A[j]$. The goal of the problem is to count the number of strong inversions for a given array A. Implement an $O(n \lg n)$-time divide-and-conquer algorithm that runs in $O(n \lg n)$ time to solve the problem. You need to read from the standard input (i.e, the terminal) and output to the standard output (i.e, the screen).

- **Input format**: The first line of the input contains one positive integers n, $1 \leq n \leq 10^6$. The next n lines contain the n integers $A[1], A[2], \cdots, A[n]$; every integer is between 0 and 10^8.
- **Output format**: Just output 1 line, which is total number of strong inversions.

Input: 6 7 3 20 16 5 8	**Output**: 4	The pairs are $(7, 3), (20, 5), (20, 8), (16, 5)$.