CSE 431/531: Algorithm Analysis and Design Spring 2019

Homework 4

Instructor: Shi Li **Deadline: 4/29/2019**

Your Name: _____ Your Student ID: _____

Problems	1	2	3	Total
Max. Score	20	20	40	80
Your Score				

Problem 1 (10 points) An independent set of a graph $G = (V, E)$ is a set $U \subseteq V$ of vertices such that there are no edges between vertices in U. Given a graph with node weights, the maximum-weight independent set problem asks for the independent set of a given graph with the maximum total weight. In general, this problem is very hard. Here we want to solve the problem on trees: given a tree with node weights, find the independent set of the tree with the maximum total weight. For example, the maximum-weight independent set of the tree in Figure 1 has weight 47.

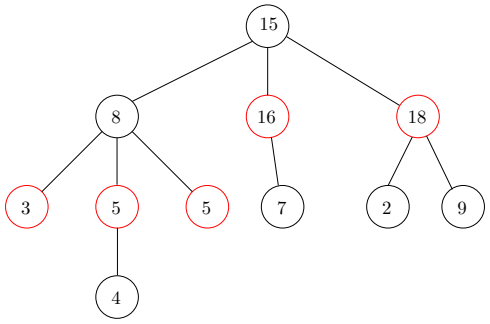

Figure 1: The maximum-weight indpendent set of the tree has weight 47. The red vertices give the independent set.

Design an $O(n)$-time algorithm for the problem, where n is the number of vertices in the tree. We assume that the nodes of the tree are $\{1, 2, 3, \cdots, n\}$. The tree is rooted at vertex 1, and for each vertex $i \in \{2, 3, \cdots, n\}$, the parent of i is a vertex $j < i$. In the input, we specify the weight w_i for each vertex $i \in \{1, 2, 3, \cdots, n\}$ and the parent of i for each $i \in \{2, 3, \cdots, n\}$.

Problem 2 (20 points) Given an array A of n numbers, we say that a 10-tuple $(i_1, i_2, \cdots, i_{10})$ of integers is inverted if $1 \leq i_1 < i_2 < i_3 < \cdots < i_{10} \leq n$ and $A[i_1] > A[i_2] > A[i_3] > \cdots > A[i_{10}]$. (Remark: If we change 10 to 2, then an inverted 2-tuple is an inversion.)

Give an $O(n^2)$-time algorithm to count the number of inverted 10-tuples w.r.t A.

Problem 3 (40 points) You need to implement the dynamic programming algorithm for the longest palindrome subsequence problem.

- **Input format**: You need to read the input from the standard input. It has one string in a line, which only contains upper and lower case letters and numbers. The length of the string is at most 2000.
- **Output format**: You need to output 2 lines to the standard output. The first line contains a number, which is length of the longest palindrome subsequence of the string. The second line contains the actual longest palindrome.

Input:	**Output**:
badccbda	6 adccda