CSE 431/531: Algorithm Analysis and Design — Spring 2020

Homework 1

Instructor: Shi Li — **Deadline: 2/17/2020**

Your Name: ______________________ Your Student ID: ________________

Problems	1	2	3	Total
Max. Score	10	16	24	50
Your Score				

Problem 1 (10 points). For each pair of functions f and g in the following table, indicate whether $f = O(g)$, $f = \Omega(g)$ and $f = \Theta(g)$ respectively. Justify your answer for the question "whether $\lceil\sqrt{10n+100}\rceil = O(n)$?", using the definition of the O-notation.

$f(n)$	$g(n)$	O	Ω	Θ
$\log_{10} n$	$\log_2(n^3)$			
$\lceil\sqrt{10n+100}\rceil$	n			
$n^3 - 100n$	$10n^2 \log n$			

Problem 2 (16 points).

(2a) **(4 points).** Given an array A of n integers, we need to check if there are two integers in the array with summation equaling 0. Consider the following simple algorithm:

for $i \leftarrow 1$ to $n-1$ **do**
    for $j \leftarrow i+1$ to n **do**
        if $A[i] + A[j] = 0$ **then return** yes
return no.

Give a tight upper bound on the running time of the algorithm.

(2b) **(12 points).** Now suppose we have the same problem as (2a) except that the array A is sorted in non-decreasing order. Consider the following algorithm:

$i \leftarrow 1, j \leftarrow n$
while $i < j$ **do**
    if $A[i] + A[j] = 0$ **then return** yes
    if $A[i] + A[j] < 0$ **then** $i \leftarrow i+1$ **else** $j \leftarrow j-1$
return no

Briefly argue about the correctness of the algorithm and give a tight upper bound on the running time of the algorithm.

Problem 3 (24 points).

(3a) **(12 points).** A cycle in an *undirected* graph $G = (V, E)$ is a sequence of $t \geq 3$ *different* vertices $v_1, v_2, \cdots, v_t$ such that $(v_i, v_{i+1}) \in E$ for every $i = 1, 2, \cdots, t-1$ and $(v_t, v_1) \in E$. Given the linked-list representation of an undirected graph $G = (V, E)$, design an $O(n+m)$-time algorithm to decide if G contains a cycle or not.

(3b) **(12 points).** A cycle in a *directed* graph $G = (V, E)$ is a sequence of $t \geq 2$ *different* vertices $v_1, v_2, \cdots, v_t$ such that $(v_i, v_{i+1}) \in E$ for every $i = 1, 2, \cdots, t-1$ and $(v_t, v_1) \in E$. Given the linked-list representation of a directed graph $G = (V, E)$, design an $O(n+m)$-time algorithm to decide if G contains a cycle or not.

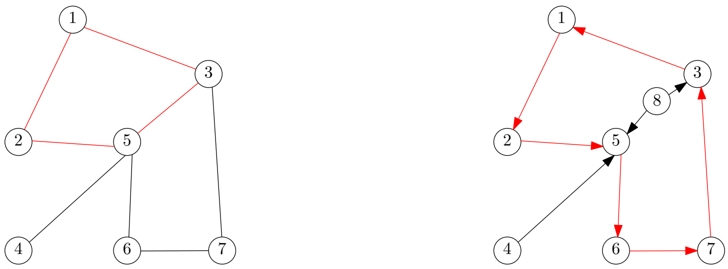

Figure 1: Cycles in undirected and directed graphs. (1, 2, 5, 3) is a cycle in the undirected graph. (1, 2, 5, 6, 7, 3) is a cycle in the directed graph. However, (1, 2, 5, 8, 3) is not a cycle in the directed graph.

Remark On a cycle of a directed graph, the directions of the edges have to be consistent. See Figure 1. So, converting a directed graph to a undirected graph and then using algorithm for (3a) does not give you a correct algorithm for (3b).