CSE 431/531: Algorithm Analysis and Design Spring 2020

Homework 2

Instructor: Shi Li **Deadline: 3/4/2020**

Your Name: _____ Your Student ID: _____

Problems	1	2	3	Total
Max. Score	10	20	20	50
Your Score				

Problem 1 (10 points). Construct the Huffman code (i.e, the optimum prefix code) for the alphabet $\{\mathsf{a}, \mathsf{b}, \mathsf{c}, \mathsf{d}, \mathsf{e}, \mathsf{f}, \mathsf{g}\}$ with the following frequencies:

Symbols	a	b	c	d	e	f	g
Frequencies	50	10	38	25	55	90	5

What is the weighted length of the code (i.e, the sum over all symbols the frequency of the symbol times its encoding length)?

Problem 2 (20 points). Let $I = (k, n, T, (p_1, p_2, \cdots, p_k), (r_1, r_2, \cdots, r_T))$ be an offline caching instance with initial set of pages, where

- k is the number of pages the cache can hold,
- n is the number of different pages,
- T is the length of the request sequence,
- $p_1, p_2, \cdots, p_k \in [n] := \{1, 2, 3, \cdots, n\}$ are the k pages in the cache initially (for simplicity we assume the k pages are all different and there are no empty pages),
- $r_t \in [n]$ for every $t \in [T]$ is the page requested at time t.

Let $I' = (k, n, T, (p_1, p_2, \cdots, p_{k-1}, p'_k), (r_1, r_2, \cdots, r_T))$ be the instance obtained from I by changing p_k to p'_k. Prove the minimum number of misses we can achieve for the instance I' is at most that for the instance I plus 1.

Problem 3 (20 points). Given a set of n points $X = \{x_1, x_2, \cdots, x_n\}$ on the real line, we want to use the smallest number of unit-length closed intervals to cover all the points in X. For example, the points X in Figure 1 can be covered by 3 unit-length intervals.

Suppose our greedy strategy is to choose some unit-length interval, and include it in the optimal solution. Which unit-length interval do you want to choose? Give your strategy and prove that it is safe to include it in the solution.

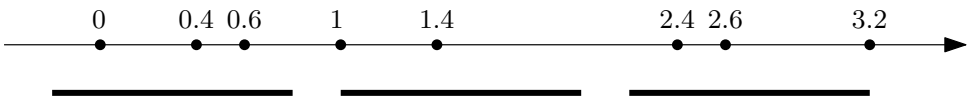

Figure 1: Using 3 unit-length intervals (denoted by thick lines) to cover points in X (denoted by the solid circles).