CSE 431/531: Algorithm Analysis and Design	Spring 2020

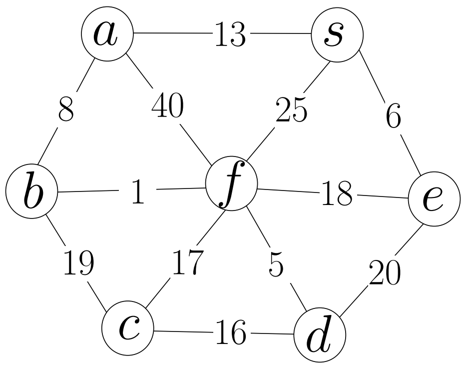
Homework 5

Instructor: Shi Li **Deadline: 5/4/2020**

Your Name: ______________________ Your Student ID: ________________

Problems	1	2	3	Total
Max. Score	20	10	20	50
Your Score				

Problem 1 (20 points). Consider the following graph G with non-negative edge weights.



(1a) Use Prim's algorithm to compute the minimum spanning tree of G. Give the minimum spanning tree and its weight.

You can use the following table to describe the execution of the algorithm. The algorithm maintains a set S of vertices. The d value of a vertex $v \notin S$ is $\min_{u \in S:(u,v)\in E} w(u,v)$. The π value of a vertex v is the vertex $u \in S$ such that $d(v) = w(u,v)$; if $d(v) = \infty$, then $\pi(v) =$ "/".

iteration	vertex added to S	a		b		c		d		e		f	
		d	π	d	π	d	π	d	π	d	π	d	π
1	s	13	s	∞	/	∞	/	∞	/	6	s	25	s
2													
3													
4													
5													
6													
7													

Table 1: Prim's Algorithm for Minimum Spanning Tree

(1b) Use Dijkstra's algorithm to compute the shortest paths from s to all other vertices in the following undirected graph.

You can use the following table to describe the execution of the algorithm. The algorithm maintains a set S of vertices. The d value of a vertex $v \notin S$ is $\min_{u \in S:(u,v)\in E} (d(u) + w(u,v))$. The π value of a vertex v is the vertex $u \in S$ such that $d(v) = d(u) + w(u,v)$; if $d(v) = \infty$, then $\pi(v) =$ "/".

iteration	vertex added to S	a		b		c		d		e		f	
		d	π	d	π	d	π	d	π	d	π	d	π
1	s	13	s	∞	/	∞	/	∞	/	6	s	25	s
2													
3													
4													
5													
6													
7													

Table 2: Dijkstra's algorithm for Shortest Path

Problem 2 (10 points) Assume we are given an undirected graph $G = (V, E)$ with non-negative edge weights $(w_e)_{e\in E}$, and two vertices s and t in V.

(2a) Let T be the unique minimum spanning tree of G. Is the following statement true or false? If we change the weight of every edge e from w_e to w_e^2, then T is still the unique minimum spanning tree of G. Justify your answer.

(2b) Let P be the unique shortest path from s to t. Is the following statement true or false? If we change the weight of every edge e from w_e to w_e^2, then P is still the unique shortest path from s to t. Justify your answer.

Problem 3 (20 points) We are given a directed graph $G = (V, E)$ with positive weight function: $w : E \to \mathbb{R}_{>0}$, and two vertices $s, t \in V$. Suppose we have already computed the d and π array using the Dijkastra's algorithm: $d[v]$ is the length of the shortest path from s to v, and $\pi[v]$ is the vertex before v in the path.

Show that how to use the d and π array to check if the shortest path from s to t in G is unique or not, in $O(n \log n + m)$ time.