CSE 431/531: Algorithm Analysis and Design (Spring 2020)

Greedy Algorithms

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1. Design efficient algorithms to solve problems

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Goals of algorithm design

1. Design efficient algorithms to solve problems
2. Design more efficient algorithms to solve problems

Common Paradigms for Algorithm Design

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- Greedy algorithms are often for optimization problems.
- They often run in polynomial time due to their simplicity.

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- At each step, make an irrevocable decision using a "reasonable" strategy

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- Show that the remaining task after applying the strategy is to solve a (many) smaller instance(s) of the same problem

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Def. A strategy is safe: there is always an optimum solution that agrees with the decision made according to the strategy.

Outline

Box Packing

Input: n boxes of capacities $c_1, c_2, \cdots, c_n$

m items of sizes $s_1, s_2, \cdots, s_m$

Can put at most 1 item in a box

Item j can be put into box i if $s_j \le c_i$

Output: A way to put as many items as possible in the boxes.

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Example:

- Box capacities: 60, 40, 25, 15, 12
- Item sizes: 45, 42, 20, 19, 16
- Can put 3 items in boxes: $45 \to 60, 20 \to 40, 19 \to 25$

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Designing a Reasonable Strategy for Box Packing

- Q: Take box 1. Which item should we put in box 1?
- A: The item of the largest size that can be put into the box.

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- formal proof via exchanging argument:

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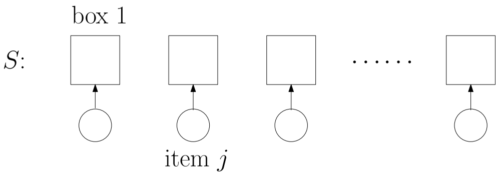

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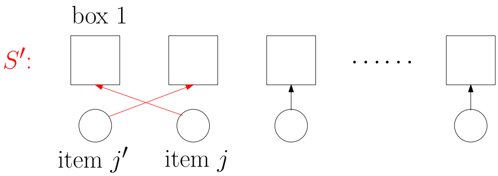

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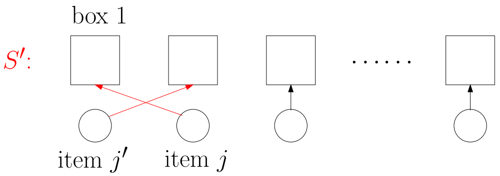

- $s_{j'} \leq s_j$, and swapping gives another solution S'
- S' is also an optimum solution. In S', j is put into Box 1. □

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- Trivial: we decided to put Item j into Box 1, and the remaining instance is obtained by removing Item j and Box 1.

Generic Greedy Algorithm

1. **while** the instance is non-trivial **do**
2.   make the choice using the greedy strategy
3.   reduce the instance

Greedy Algorithm for Box Packing

1. $T \leftarrow \{1, 2, 3, \cdots, m\}$
2. for $i \leftarrow 1$ to n do
3.   if some item in T can be put into box i, then
4. $j \leftarrow$ the largest item in T that can be put into box i
5.   print("put item j in box i")
6. $T \leftarrow T \setminus \{j\}$

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Lemma Generic algorithm is correct if and only if the greedy strategy is safe.

- Greedy strategy is safe: we will not miss the optimum solution
- Greedy stretegy is not safe: we will miss the optimum solution for some instance, since the choices we made are irrevocable.

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Def. A strategy is "safe" if there is always an optimum solution that is "consistent" with the decision made according to the strategy.

Exchange argument: Proof of Safety of a Strategy

- let S be an arbitrary optimum solution.
- if S is consistent with the greedy choice, done.
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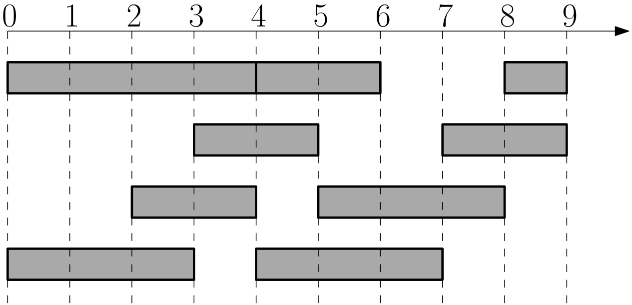
Outline

Interval Scheduling

Input: n jobs, job i with start time s_i and finish time f_i

i and j are compatible if $[s_i, f_i)$ and $[s_j, f_j)$ are disjoint

Output: A maximum-size subset of mutually compatible jobs



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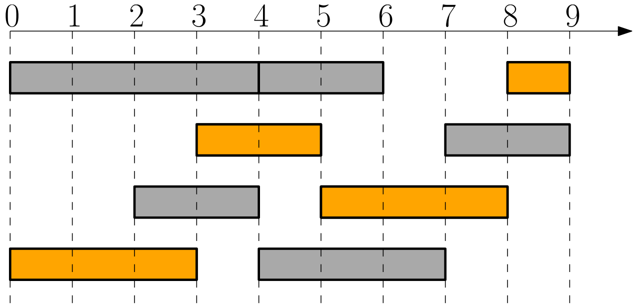

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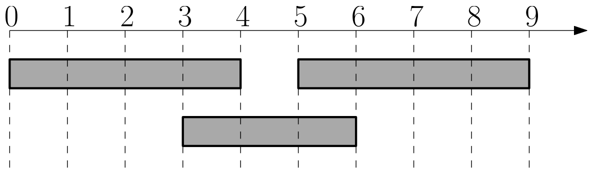

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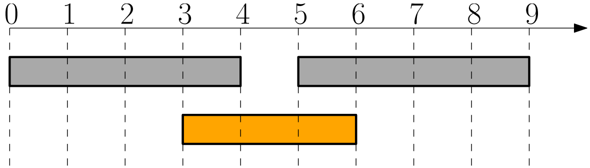

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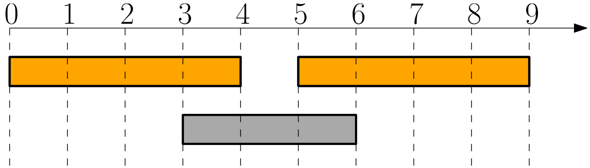

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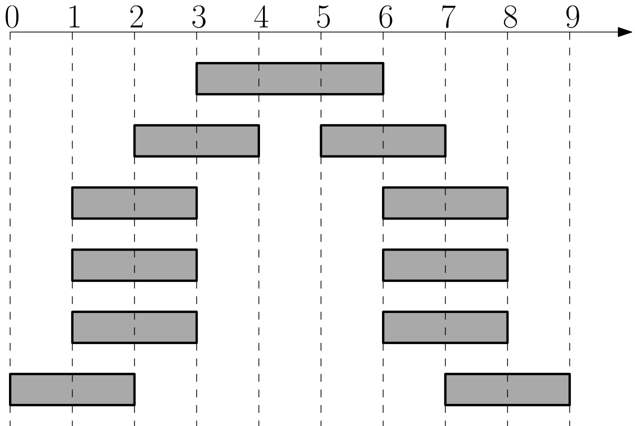

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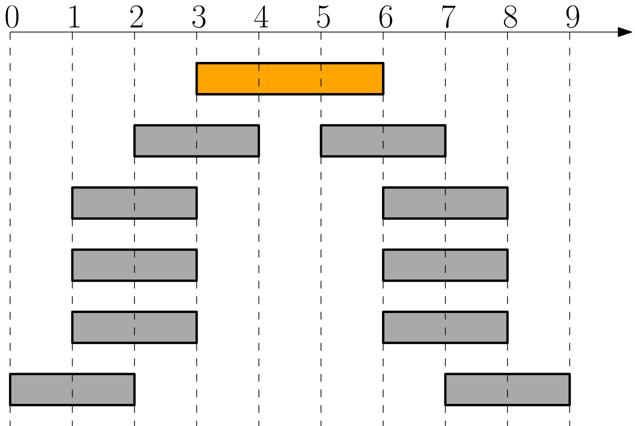

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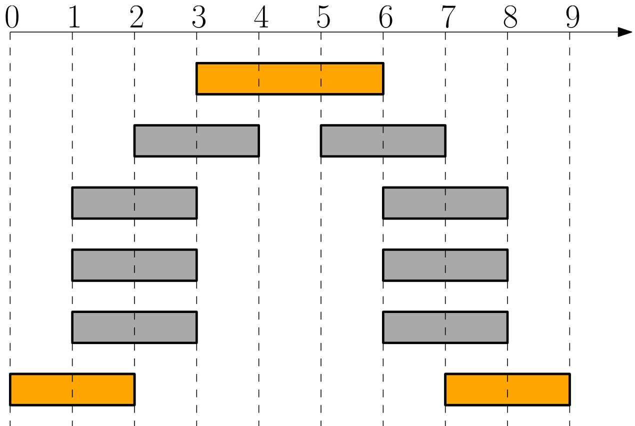

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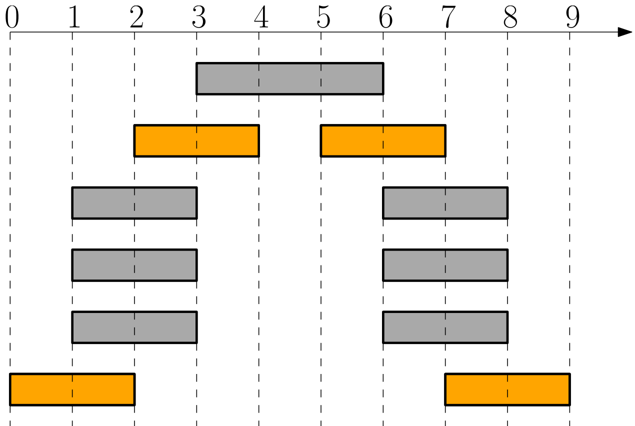

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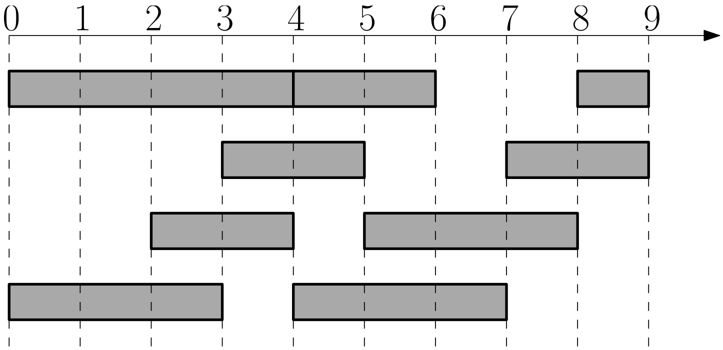

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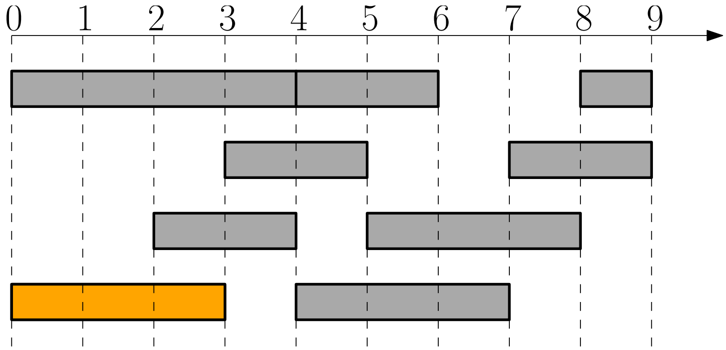

Greedy Algorithm for Interval Scheduling

Lemma It is safe to schedule the job j with the earliest finish time: There is an optimum solution where the job j with the earliest finish time is scheduled.

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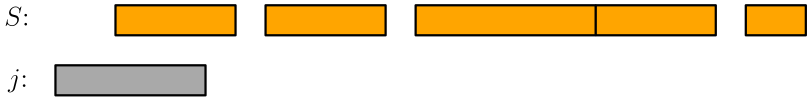

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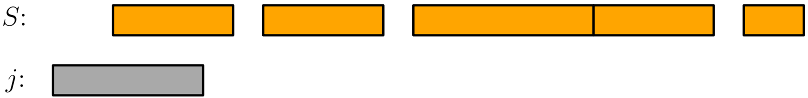

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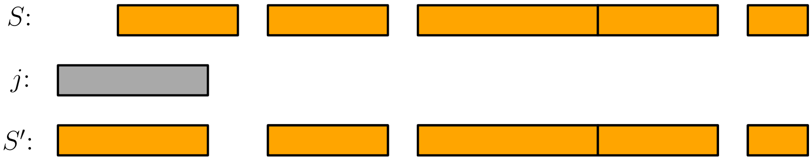

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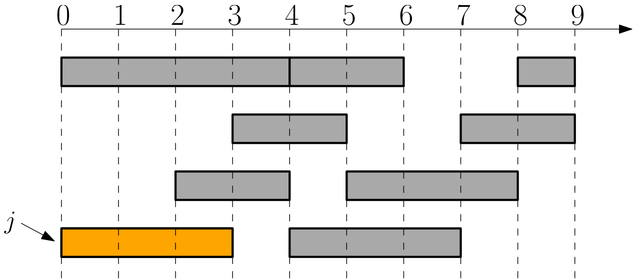
Lemma It is safe to schedule the job j with the earliest finish time: There is an optimum solution where the job j with the earliest finish time is scheduled.

- What is the remaining task after we decided to schedule j?
- Is it another instance of interval scheduling problem?



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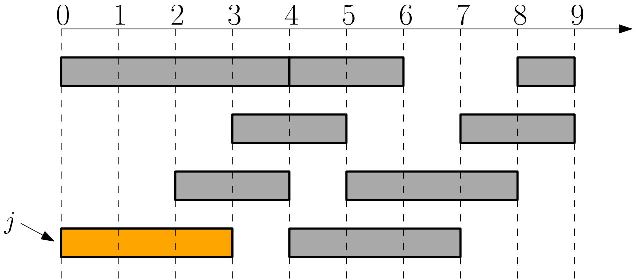

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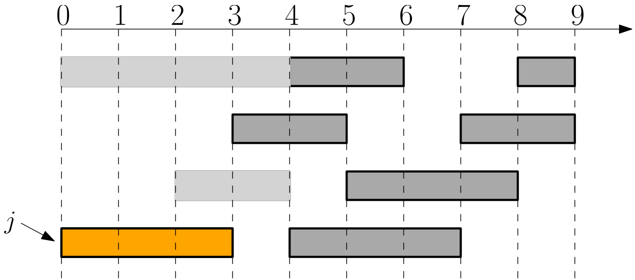

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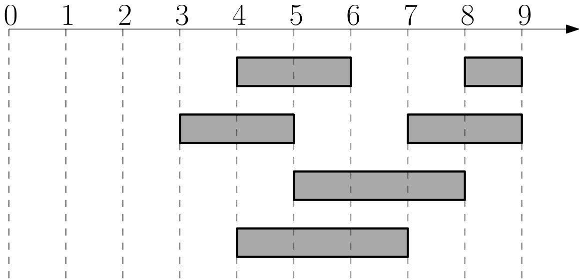

Greedy Algorithm for Interval Scheduling

Schedule(s, f, n)

1. $A \leftarrow \{1, 2, \cdots, n\}, S \leftarrow \emptyset$
2. while $A \neq \emptyset$
3. $\quad j \leftarrow \arg\min_{j' \in A} f_{j'}$
4. $\quad S \leftarrow S \cup \{j\}$; $A \leftarrow \{j' \in A : s_{j'} \geq f_j\}$
5. return S

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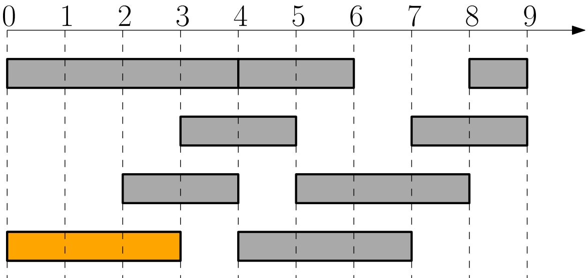

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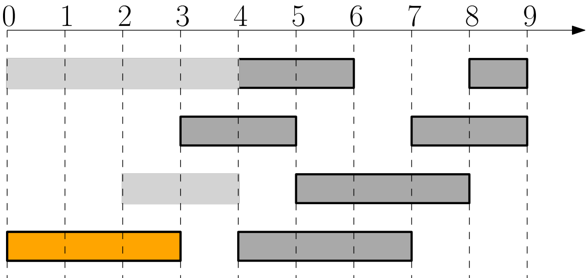

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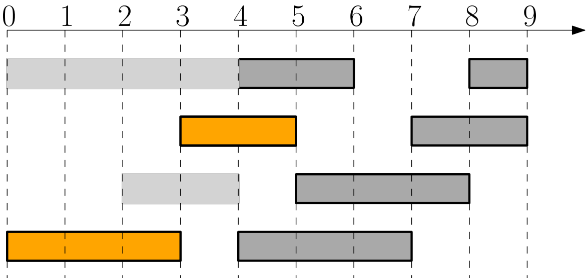

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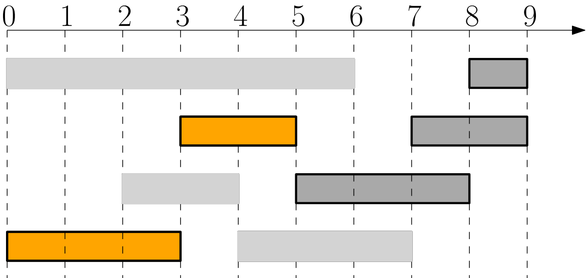
1. $A \leftarrow \{1, 2, \cdots, n\}, S \leftarrow \emptyset$
2. while $A \neq \emptyset$
3. $\quad j \leftarrow \arg\min_{j' \in A} f_{j'}$
4. $\quad S \leftarrow S \cup \{j\}$; $A \leftarrow \{j' \in A : s_{j'} \geq f_j\}$
5. return S



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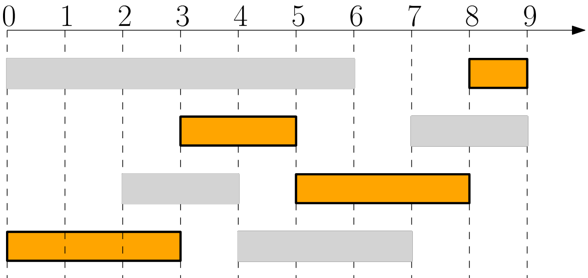

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- Naive implementation: $O(n^2)$ time

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Running time of algorithm?

- Naive implementation: $O(n^2)$ time
- Clever implementation: $O(n \lg n)$ time

Clever Implementation of Greedy Algorithm

Schedule(s, f, n)

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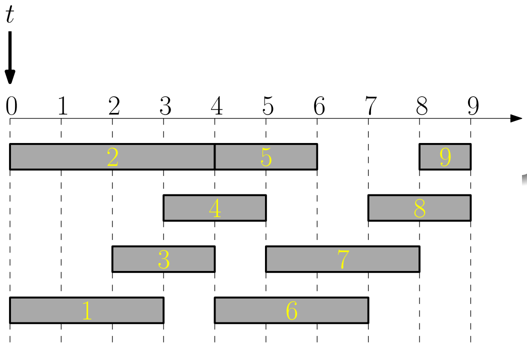

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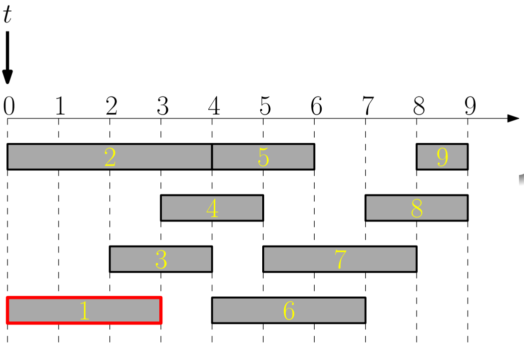

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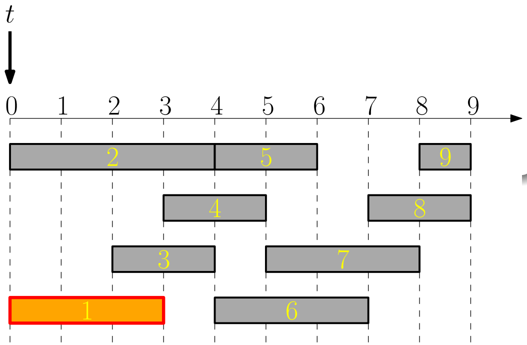

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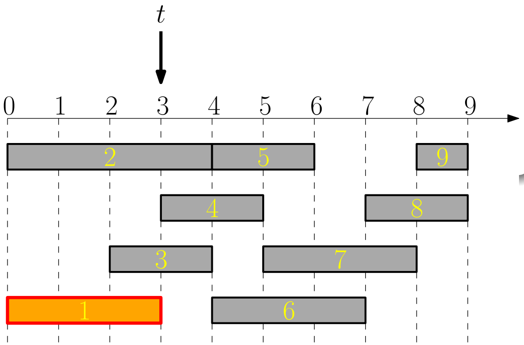

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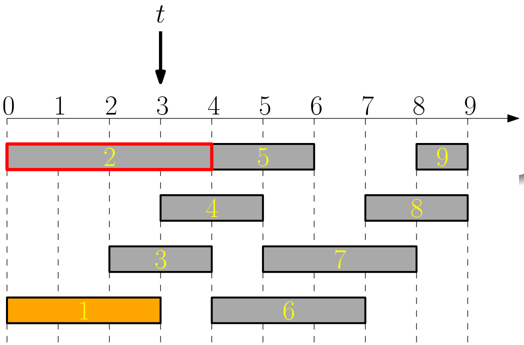

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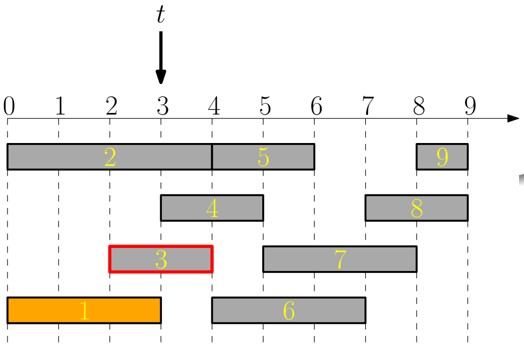

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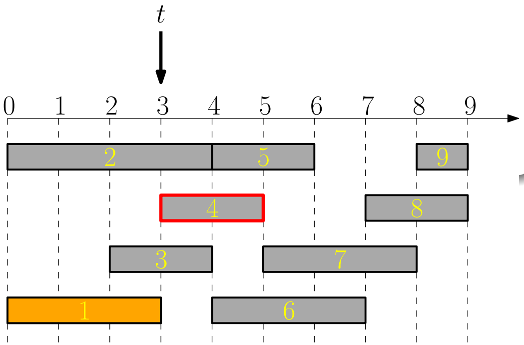

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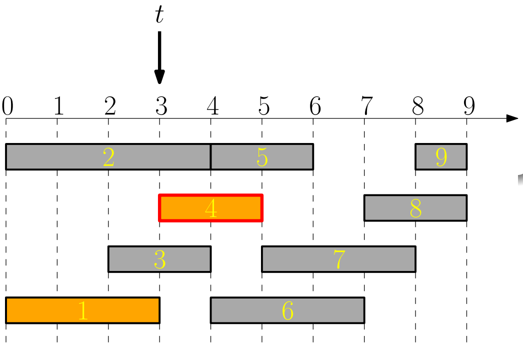

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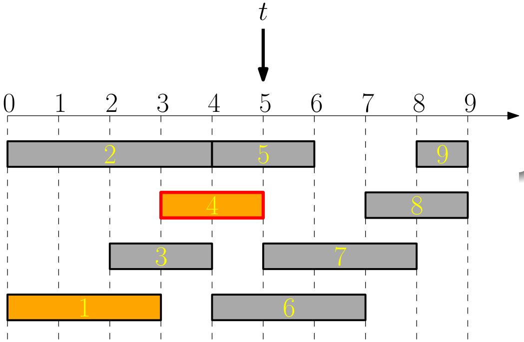

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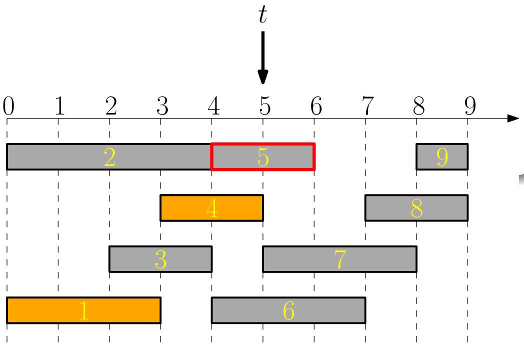

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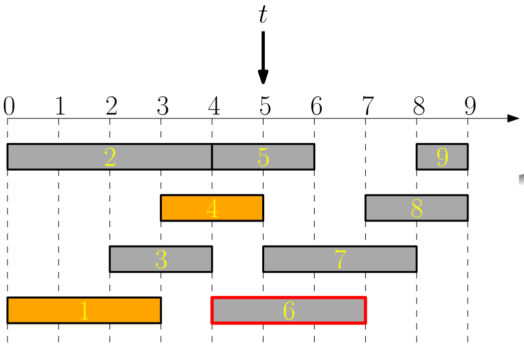

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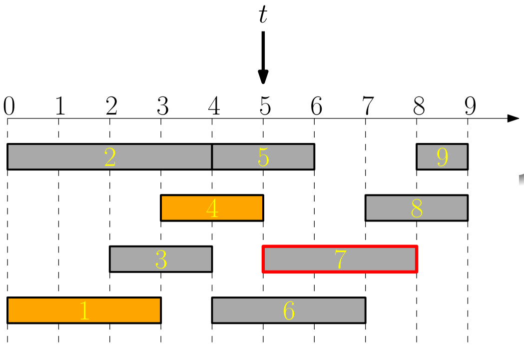

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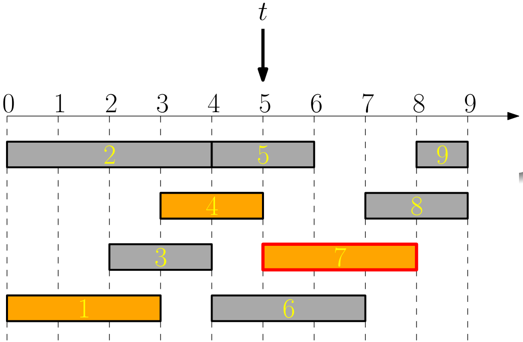

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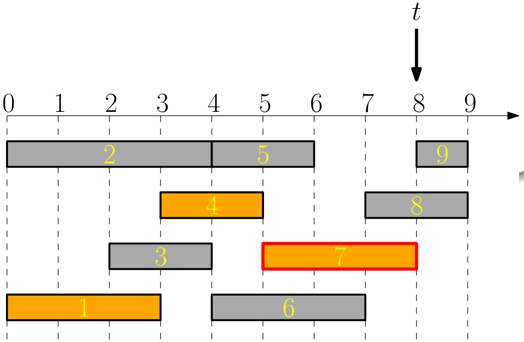

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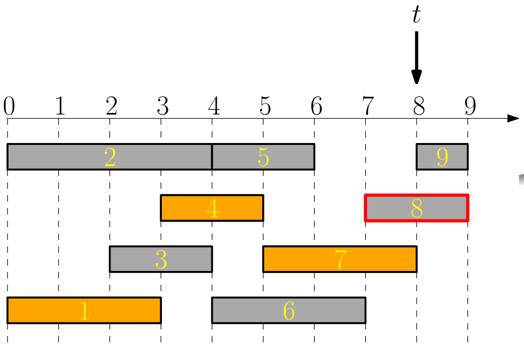

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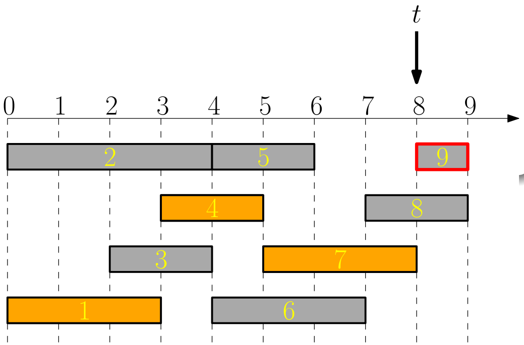

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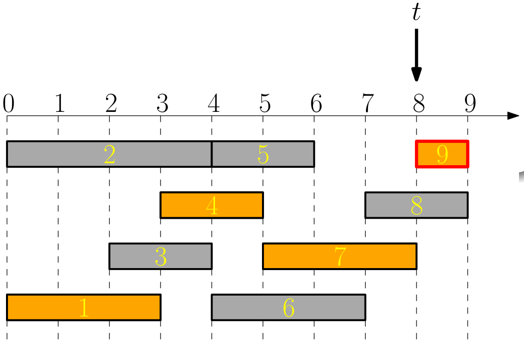

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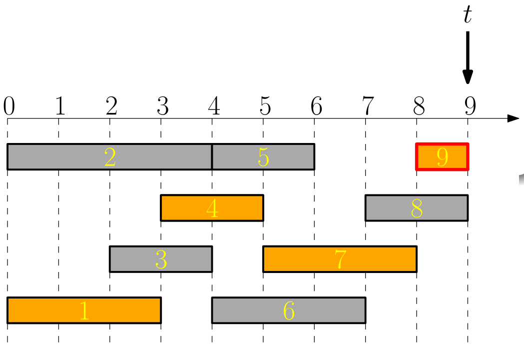

Outline

Offline Caching: Example

Offline Caching

- Cache that can store k pages
- Sequence of page requests

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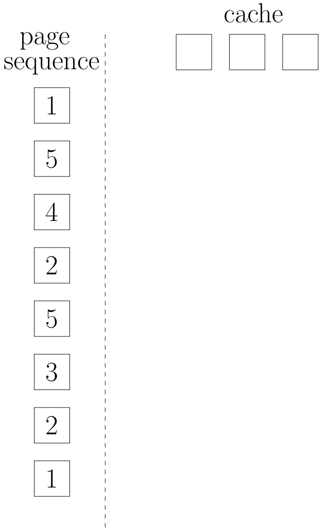

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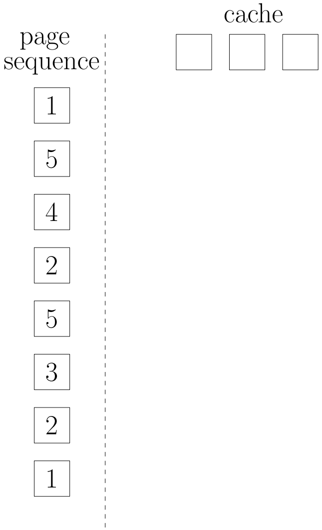

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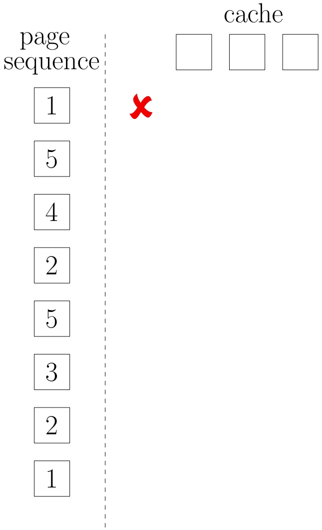

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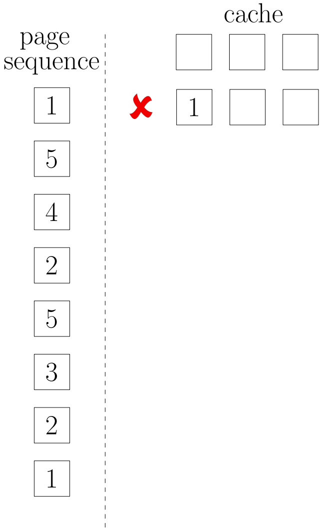

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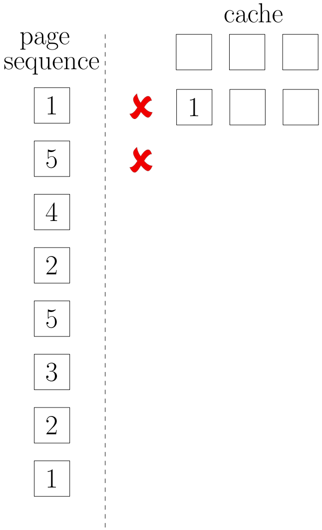

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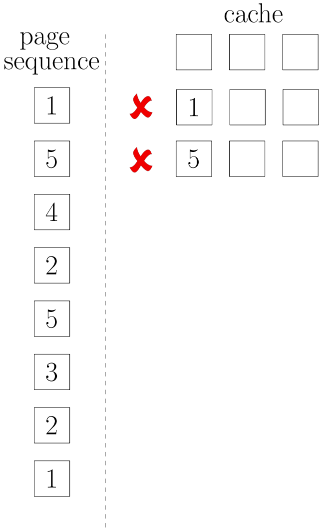

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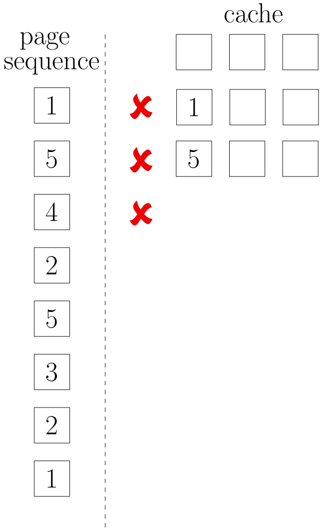

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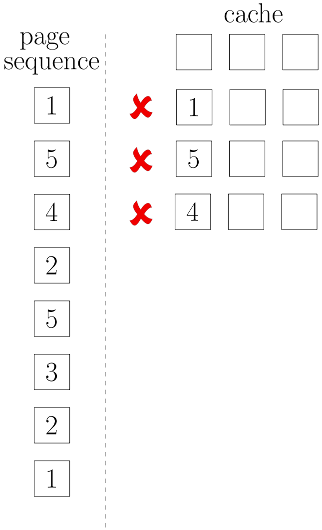

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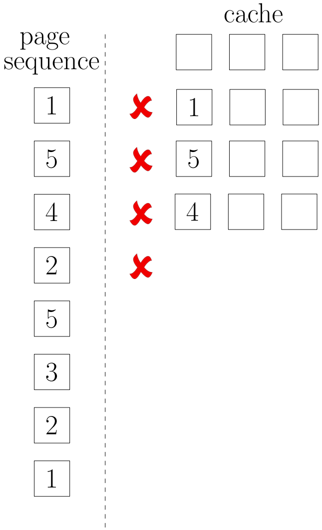

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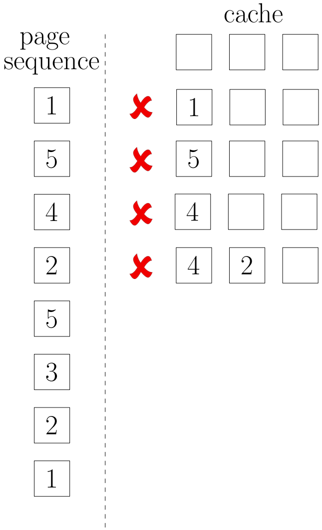

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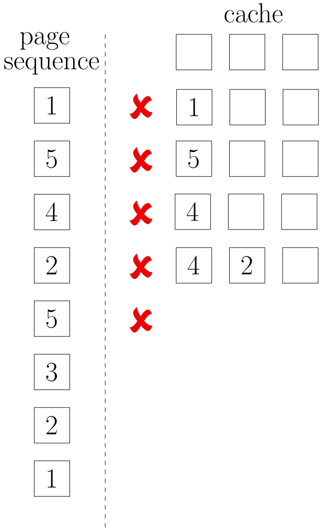

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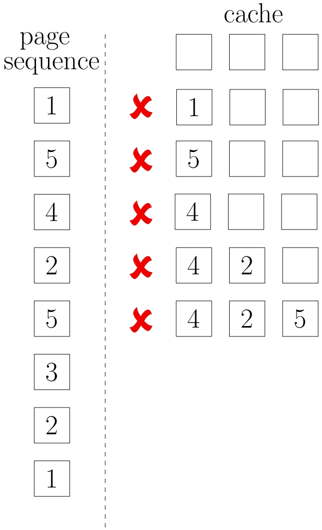

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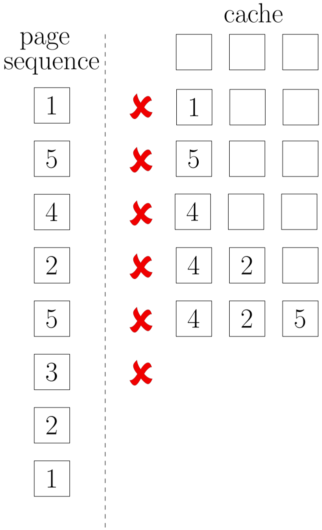

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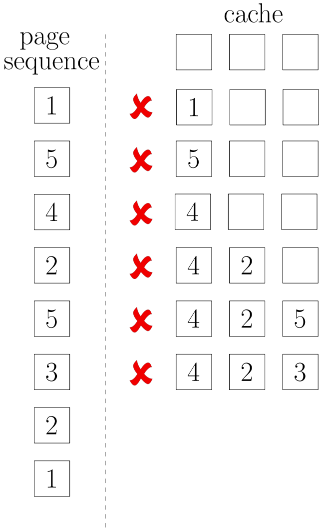

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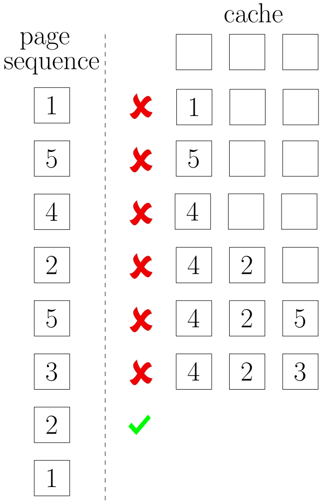

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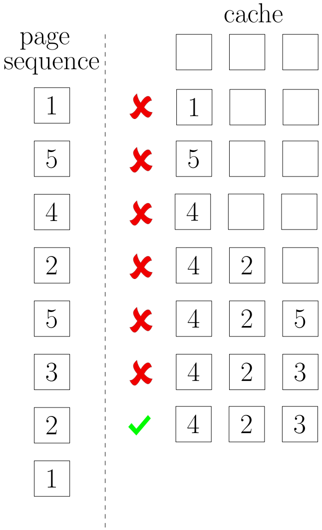

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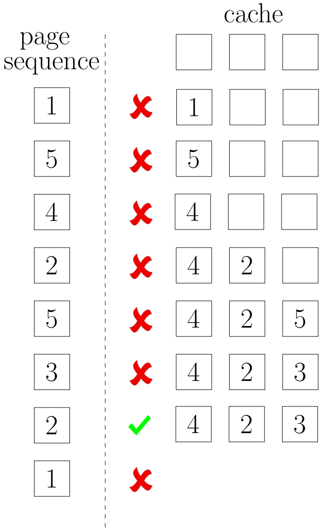

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page sequence		cache		
1	✗	1		
5	✗	5		
4	✗	4		
2	✗	4	2	
5	✗	4	2	5
3	✗	4	2	3
2	✓	4	2	3
1	✗	1	2	3

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2	✘	4	2	
5	✘	4	2	5
3	✘	4	2	3
2	✓	4	2	3
1	✘	1	2	3

misses $= 7$

Offline Caching

- Cache that can store k pages
- Sequence of page requests
- Cache miss happens if requested page not in cache. We need bring the page into cache, and evict some existing page if necessary.
- Cache hit happens if requested page already in cache.
- Goal: minimize the number of cache misses.

page sequence		cache		
1	✘	1		
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4	✘	4		
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5	✘	4	2	5
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2	✓	4	2	3
1	✘	1	2	3

misses $= 7$

A Better Solution for Example

page sequence		cache				cache		
1	✗	1			✗	1		
5	✗	5			✗	5		
4	✗	4			✗	5	4	
2	✗	4	2		✗	5	4	2
5	✗	4	2	5	✓	5	4	2
3	✗	4	2	3	✗	5	3	2
2	✓	4	2	3	✓	5	3	2
1	✗	1	2	3	✗	1	3	2
		misses $= 7$				misses $= 6$		

Offline Caching Problem

Input: k : the size of cache

n : number of pages

We use $[n]$ for $\{1, 2, 3, \cdots, n\}$.

$\rho_1, \rho_2, \rho_3, \cdots, \rho_T \in [n]$: sequence of requests

Output: $i_1, i_2, i_3, \cdots, i_T \in \{\text{hit}, \text{empty}\} \cup [n]$: indices of pages to evict ("hit" means evicting no page, "empty" means evicting empty page)

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A: Online caching

Q: Why do we study the offline caching problem?

A: Use the offline solution as a benchmark to measure the "competitive ratio" of online algorithms

Offline Caching: Potential Greedy Algorithms

- FIFO(First-In-First-Out): always evict the first page in cache

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- All the above algorithms are not optimum!
- Indeed all the algorithms are "online", i.e, the decisions can be made without knowing future requests. Online algorithms can not be optimum.

FIFO is not optimum

FIFO

requests

1

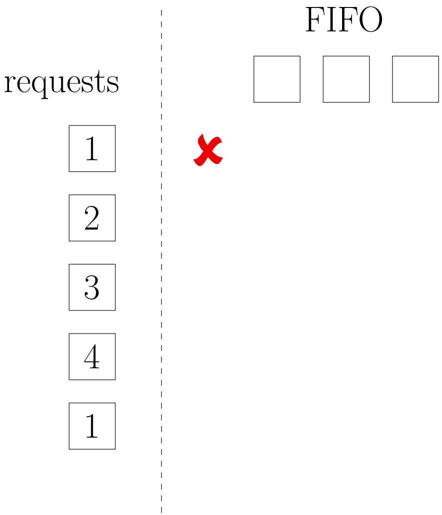
2

3

4

1

FIFO is not optimum



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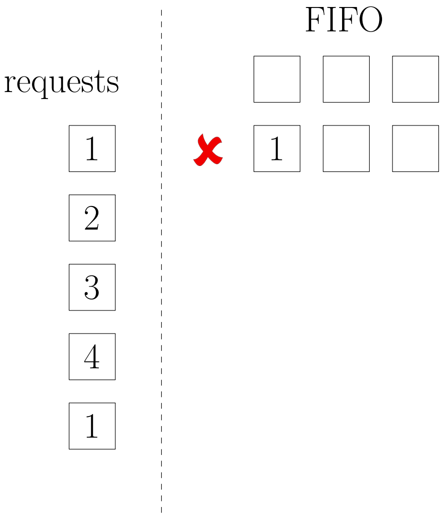

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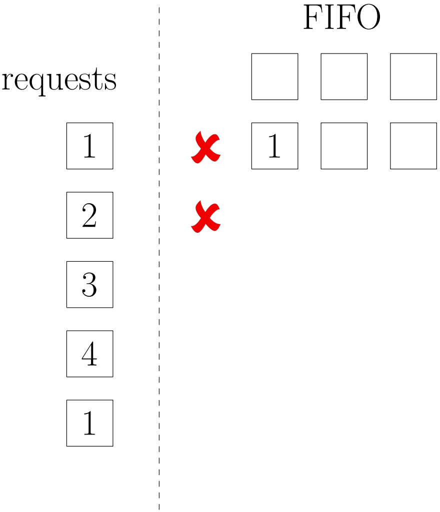

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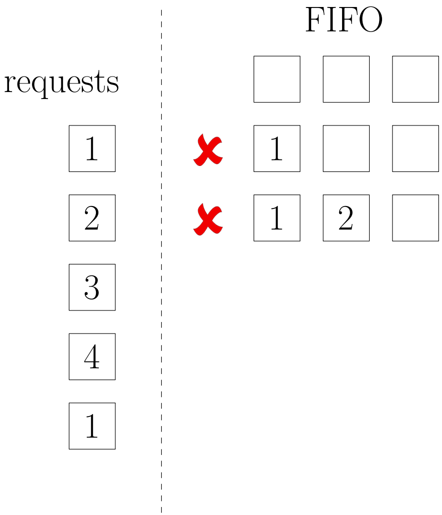

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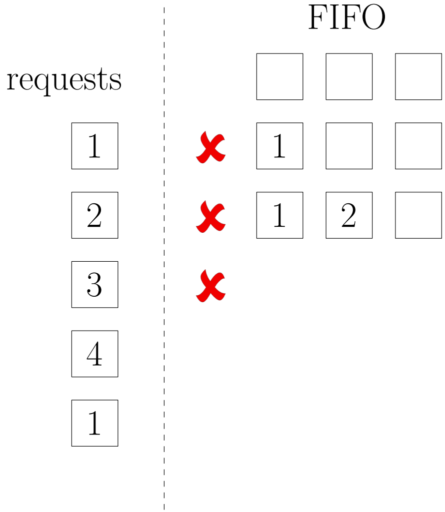

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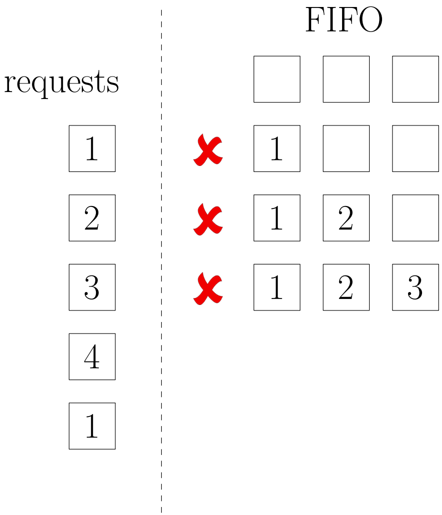

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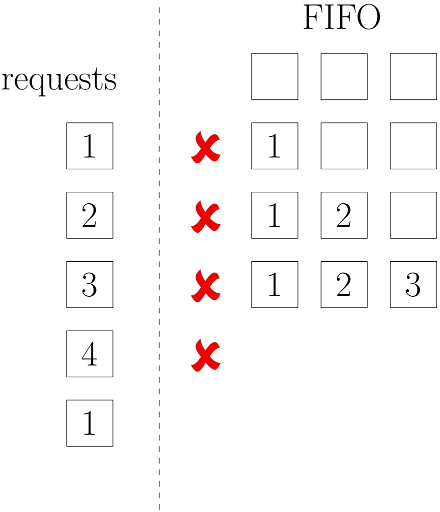

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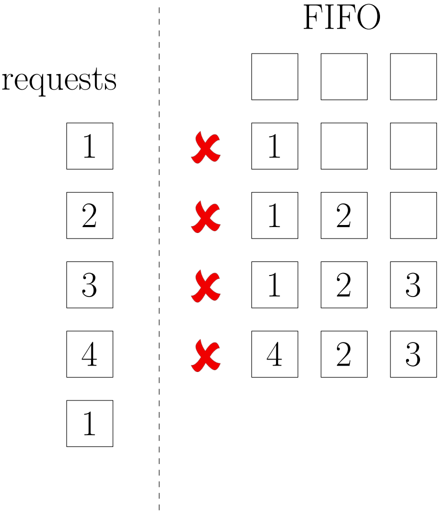

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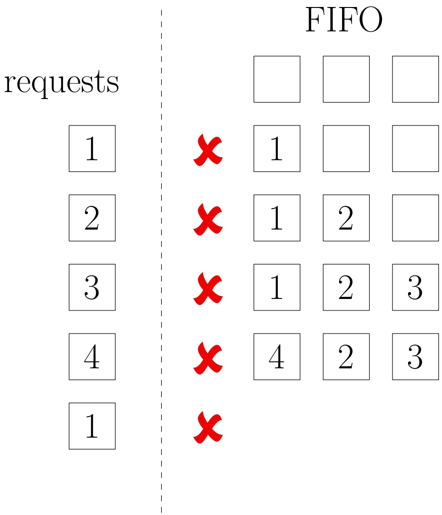

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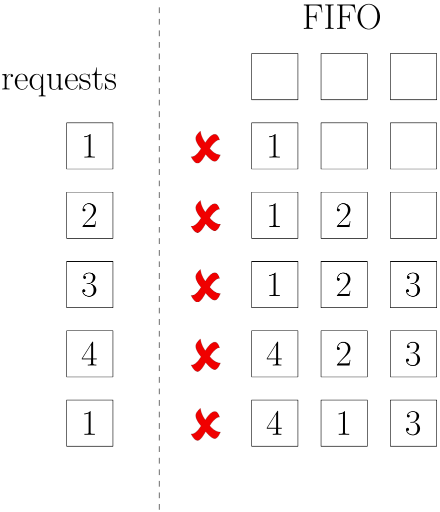


FIFO is not optimum

requests		FIFO		
1	✘	1		
2	✘	1	2	
3	✘	1	2	3
4	✘	4	2	3
1	✘	4	1	3

misses $= 5$

FIFO is not optimum

requests		FIFO				Furthest-in-Future		
1	✗	1			✗	1		
2	✗	1	2		✗	1	2	
3	✗	1	2	3	✗	1	2	3
4	✗	4	2	3	✗	1	4	3
1	✗	4	1	3	✓	1	4	3
		misses $= 5$				misses $= 4$		

Optimum Offline Caching

Furthest-in-Future (FF)

- Algorithm: every time, evict the item that is not requested until furthest in the future, if we need to evict one.
- The algorithm is not an online algorithm, since the decision at a step depends on the request sequence in the future.

Furthest-in-Future (FF)

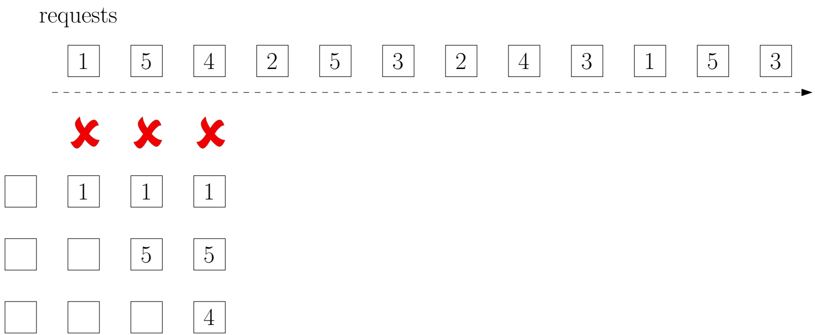
requests	FIFO				Furthest-in-Future			
1	✗	1			✗	1		
2	✗	1	2		✗	1	2	
3	✗	1	2	3	✗	1	2	3
4	✗	4	2	3	✗	1	4	3
1	✗	4	1	3	✓	1	4	3
	misses = 5				misses = 4			

Example

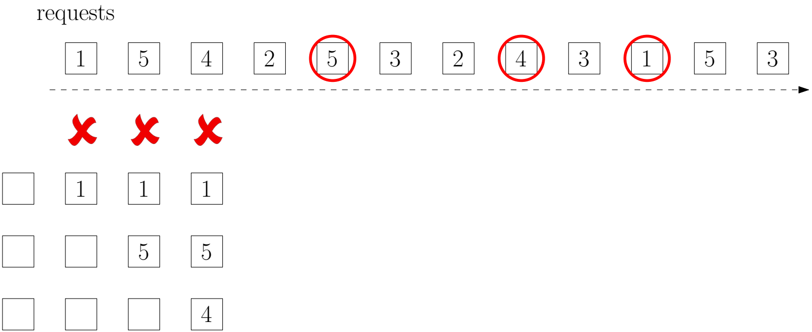
requests

1 5 4 2 5 3 2 4 3 1 5 3

Example



Example



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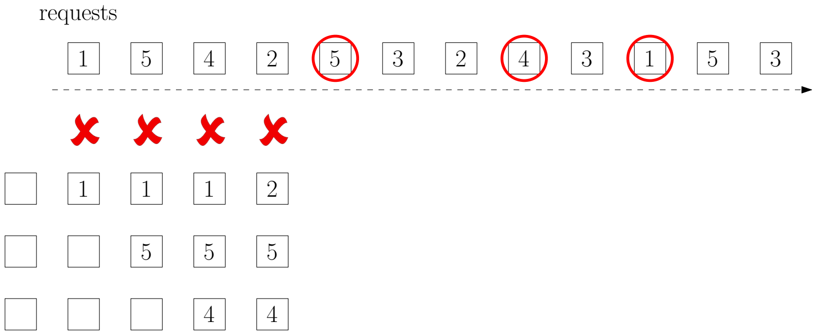
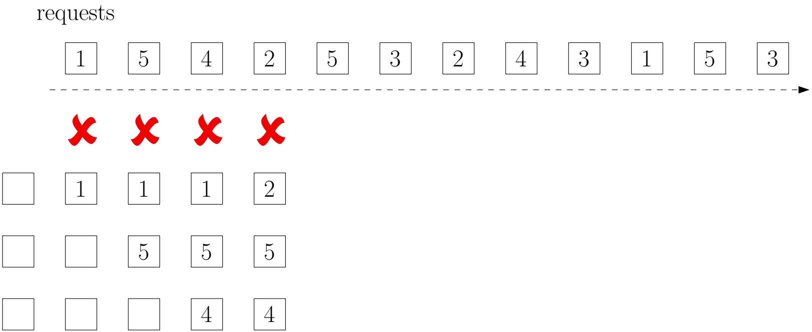


Example



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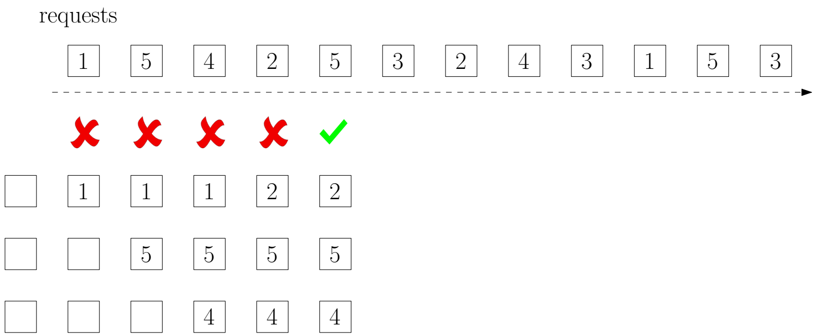

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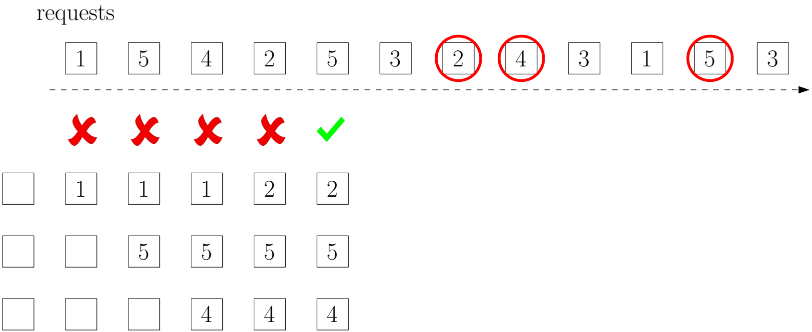

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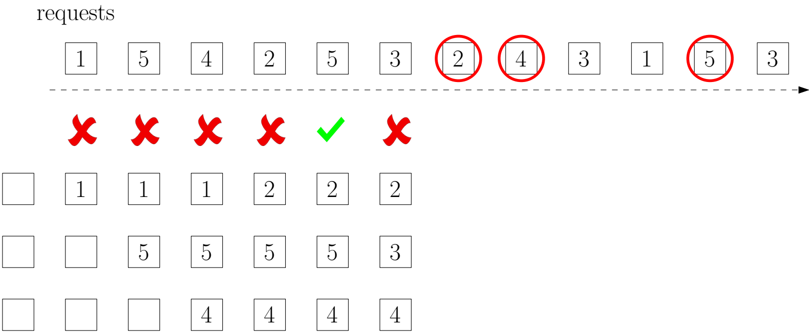

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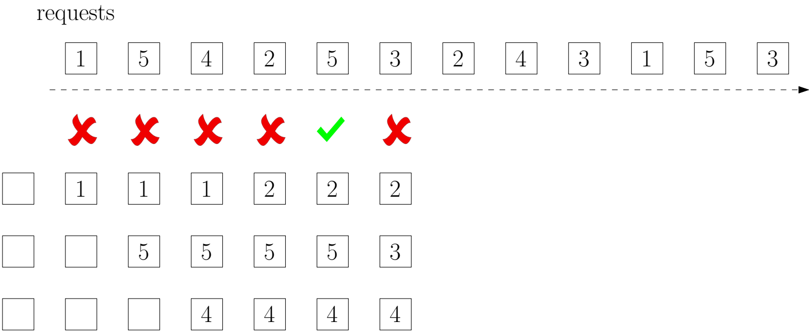

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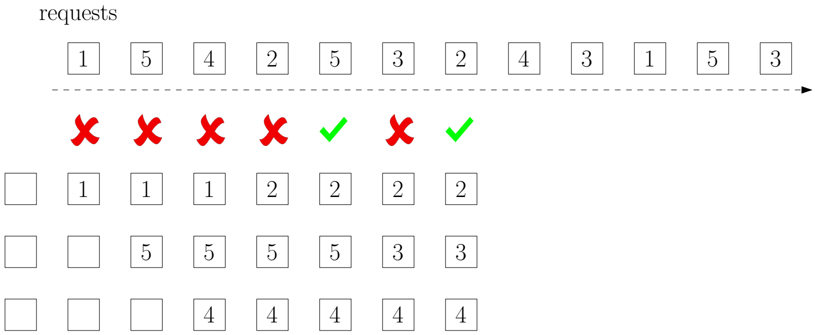

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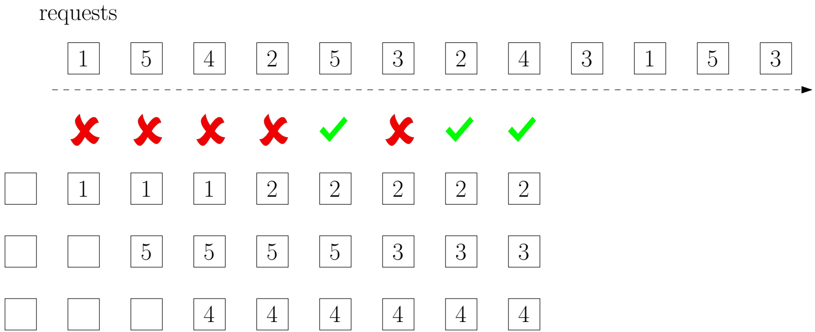

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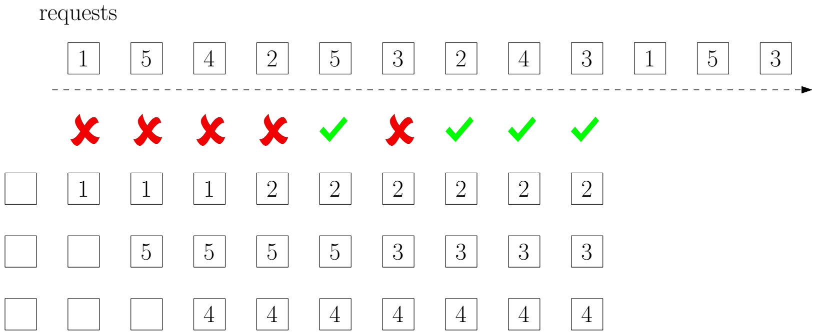

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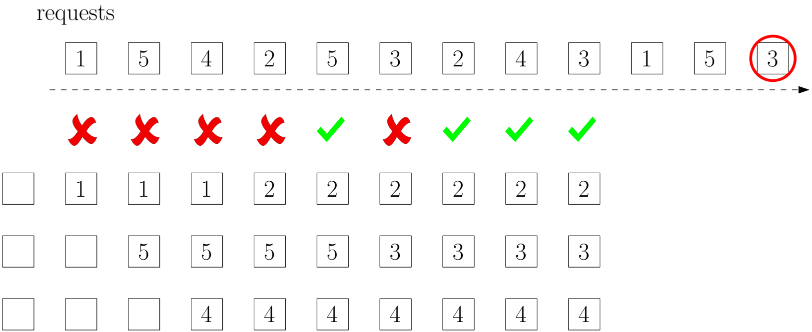
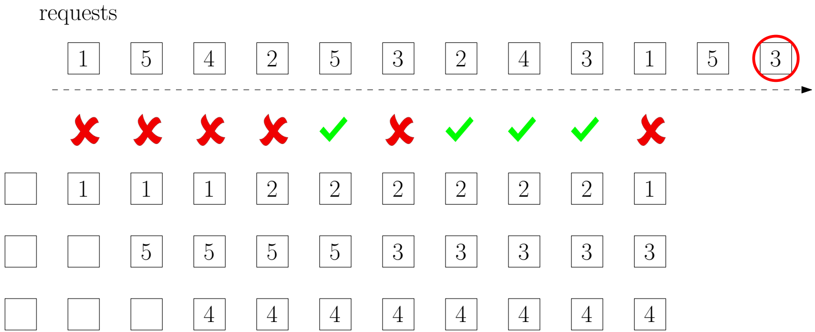


Example



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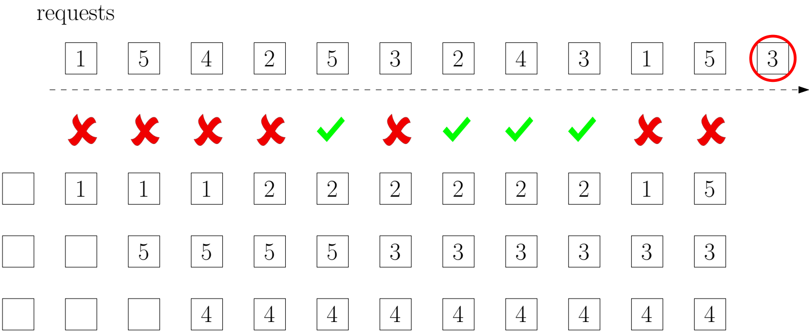
requests
1 5 4 2 5 3 2 4 3 1 5 3
1 1 1 2 2 2 2 2 2 1 5
5 5 5 5 3 3 3 3 3 3
4 4 4 4 4 4 4 4 4

Example

requests

	1	5	4	2	5	3	2	4	3	1	5	3
	✗	✗	✗	✗	✓	✗	✓	✓	✓	✗	✗	✓
	1	1	1	2	2	2	2	2	2	1	5	5
		5	5	5	5	3	3	3	3	3	3	3
			4	4	4	4	4	4	4	4	4	4

Recall: Designing and Analyzing Greedy Algorithms

Greedy Algorithm

- Build up the solutions in steps
- At each step, make an irrevocable decision using a "reasonable" strategy

Analysis of Greedy Algorithm

- Prove that the reasonable strategy is "safe" (key)
- Show that the remaining task after applying the strategy is to solve a (many) smaller instance(s) of the same problem (usually easy)

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Offline Caching Problem

Input: k : the size of cache

n : number of pages

$\rho_1, \rho_2, \rho_3, \cdots, \rho_T \in [n]$: sequence of requests

Output: $i_1, i_2, i_3, \cdots, i_t \in \{\mathrm{hit}, \text{empty}\} \cup [n]$

- empty stands for an empty page
- "hit" means evicting no pages

Offline Caching Problem

Input: k : the size of cache

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$\rho_1, \rho_2, \rho_3, \cdots, \rho_T \in [n]$: sequence of requests

$p_1, p_2, \cdots, p_k \in \{\text{empty}\} \cup [n]$: initial set of pages in cache

Output: $i_1, i_2, i_3, \cdots, i_t \in \{\text{hit}, \text{empty}\} \cup [n]$

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Lemma Assume at time 1 a page fault happens and there are no empty pages in the cache. Let p^* be the page in cache that is not requested until furthest in the future. It is safe to evict p^* at time 1.

Analysis of Greedy Algorithm

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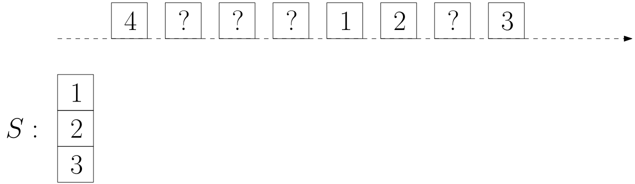
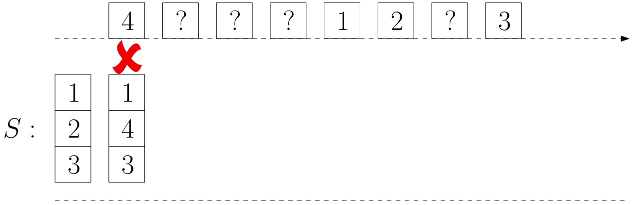
Lemma Assume at time 1 a page fault happens and there are no empty pages in the cache. Let p^* be the page in cache that is not requested until furthest in the future. There is an optimum solution in which p^* is evicted at time 1.



Proof.

1. S: any optimum solution
2. p^*: page in cache not requested until furthest in the future.
   - In the example, $p^* = 3$.



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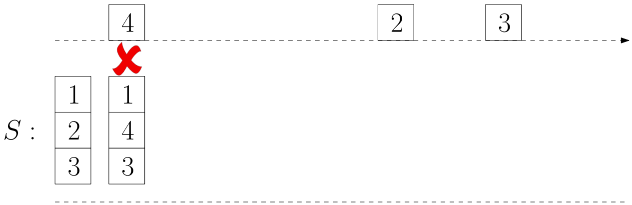

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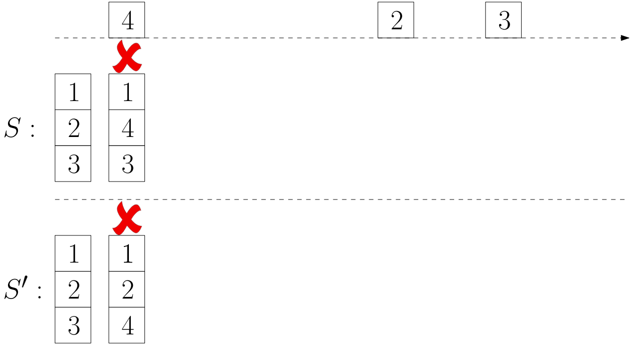
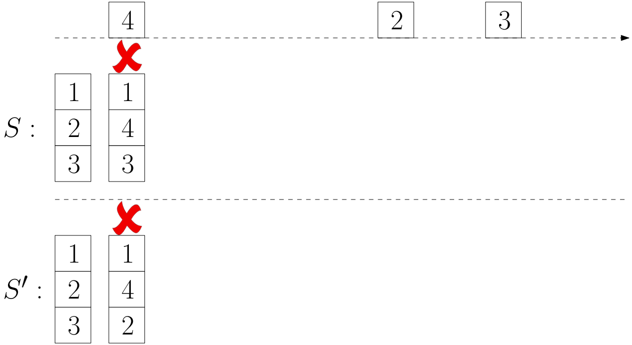


Proof.

4. Create S'. S' evicts $p^*(=3)$ instead of $p'(=2)$ at time 1.



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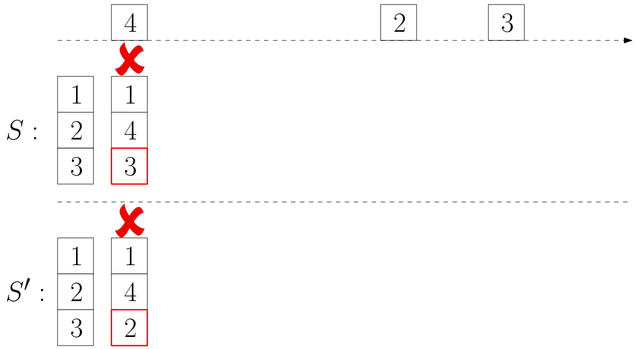

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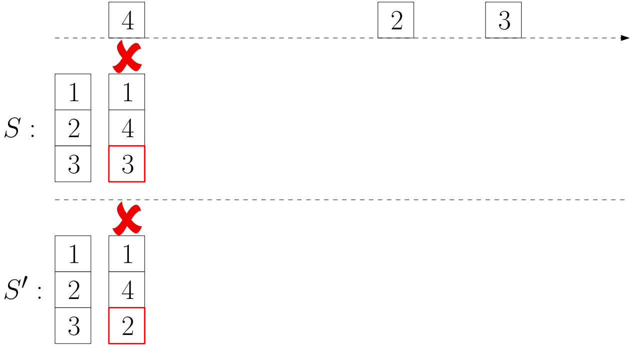

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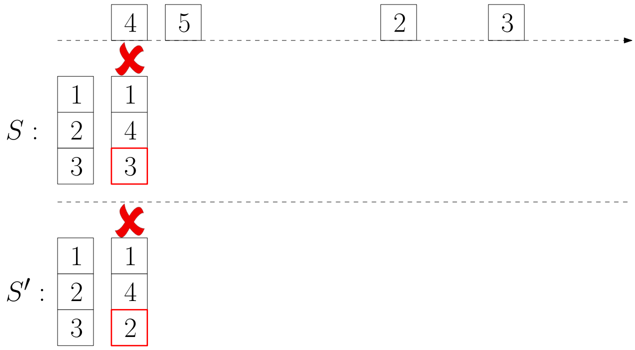

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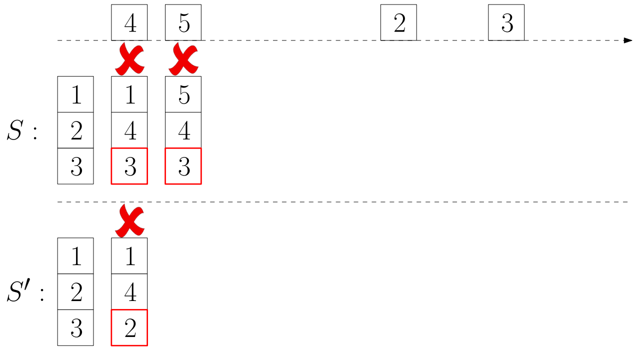

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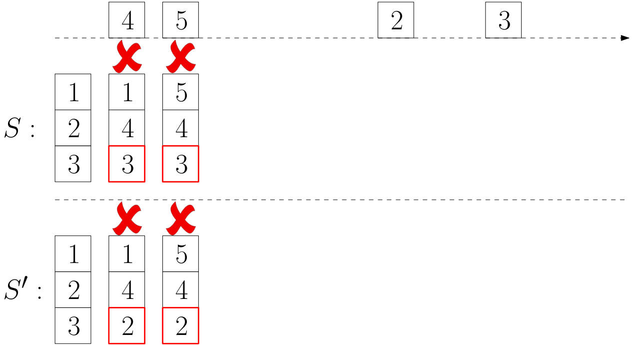

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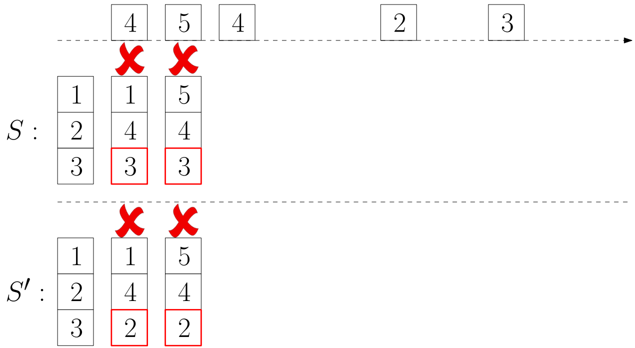

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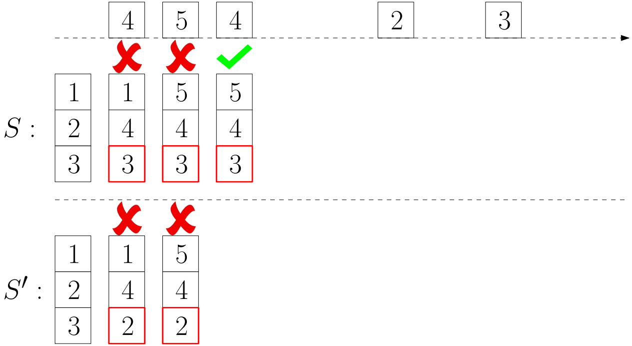

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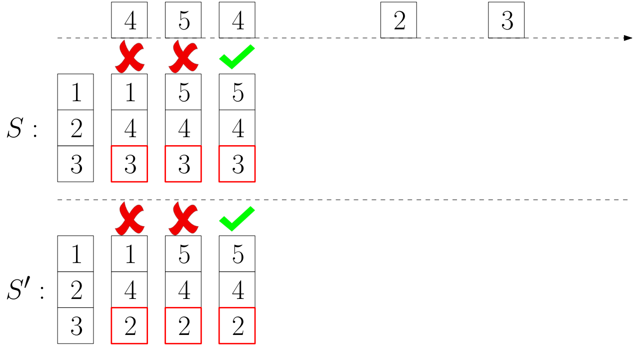

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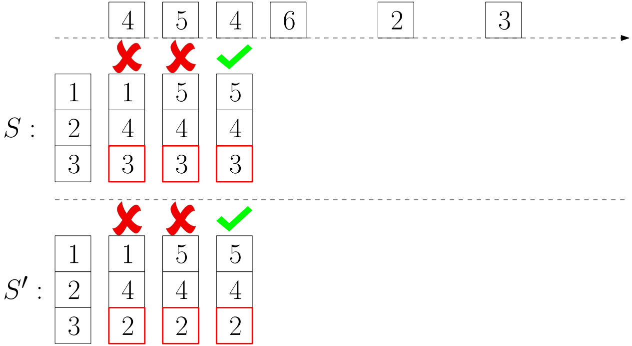

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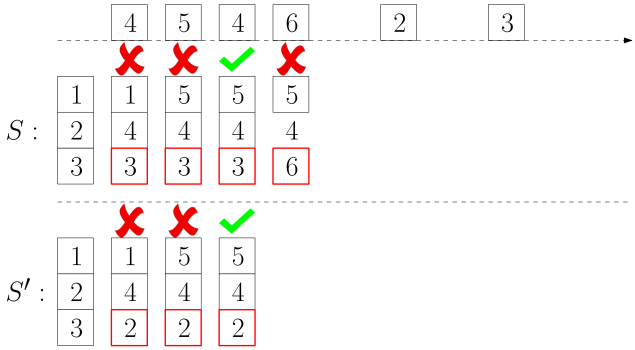

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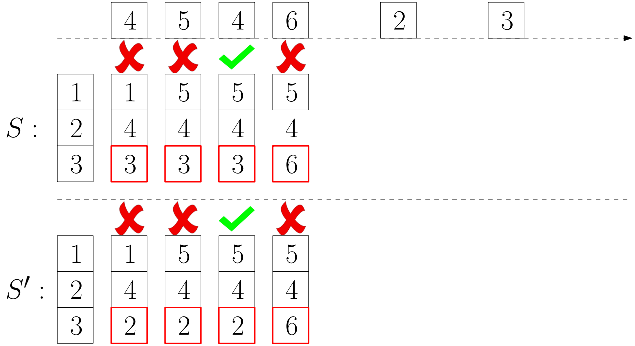

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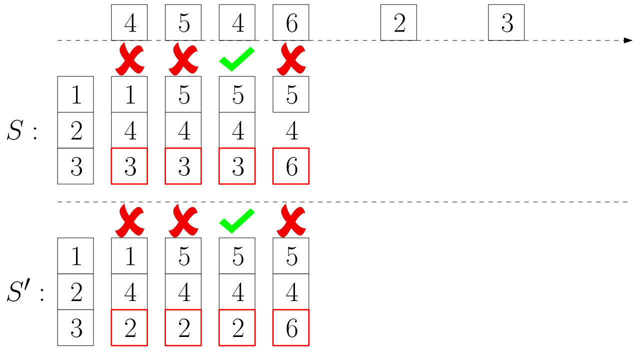
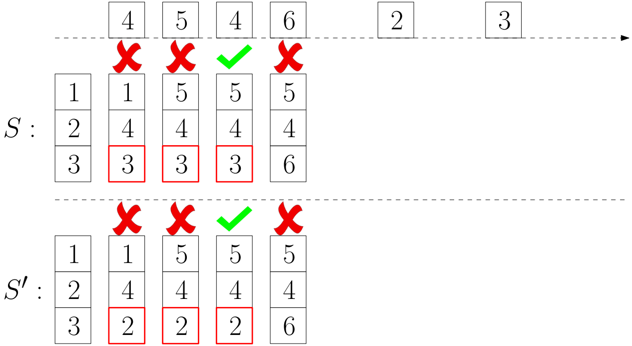
5. After time 1, cache status of S and that of S' differ by only 1 page. S contains $p'(=2)$ and S contains $p^*(=3)$.
6. From now on, S' will "copy" S.



Proof.



Proof.

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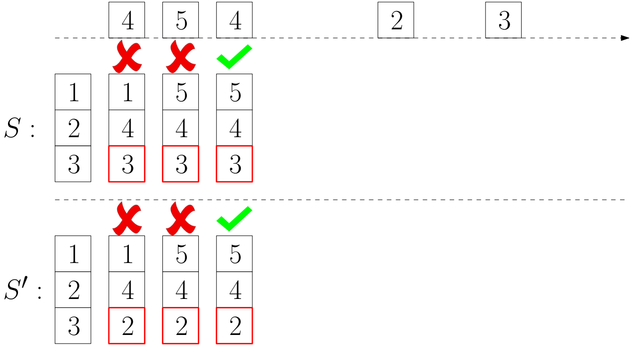

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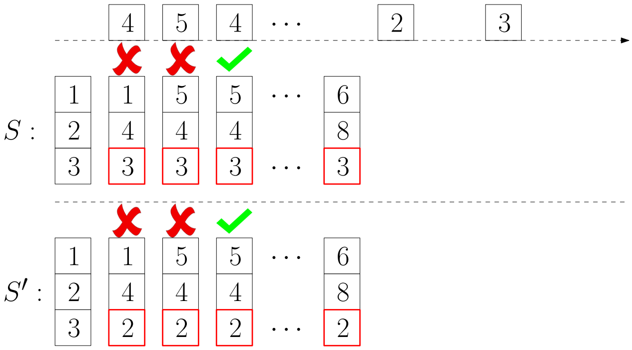

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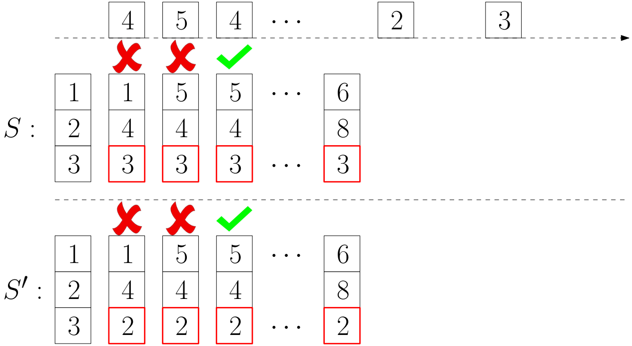

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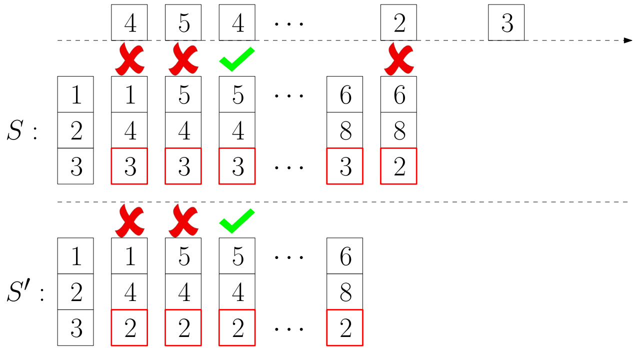

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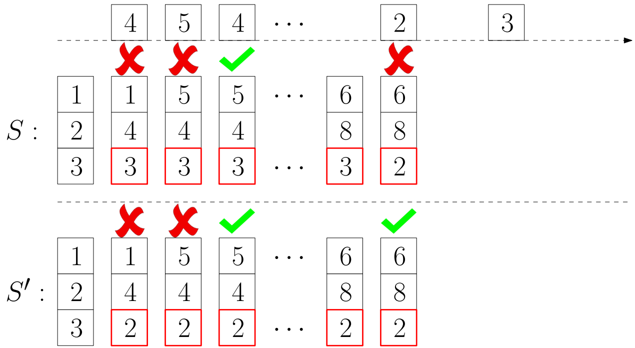

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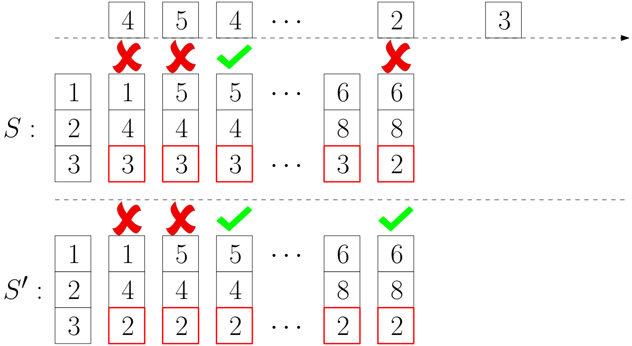

Proof.

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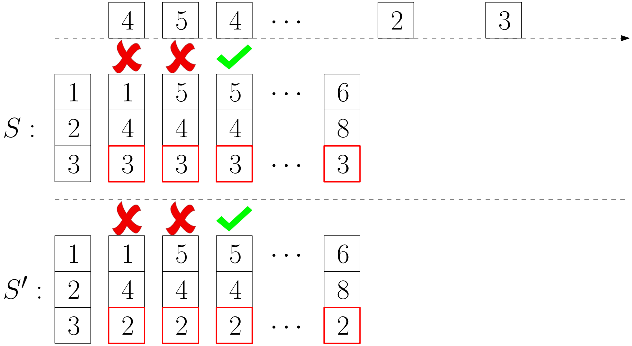

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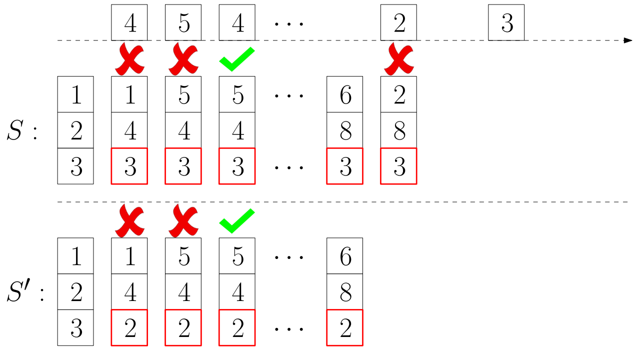

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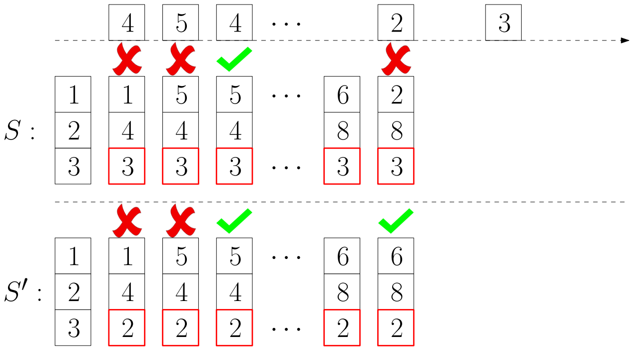

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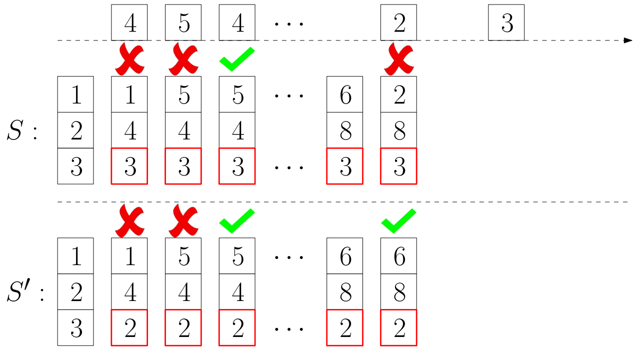

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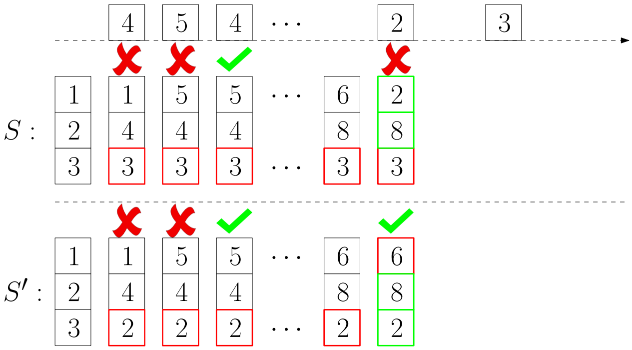
10. So far, S' has 1 less page-miss than S does.



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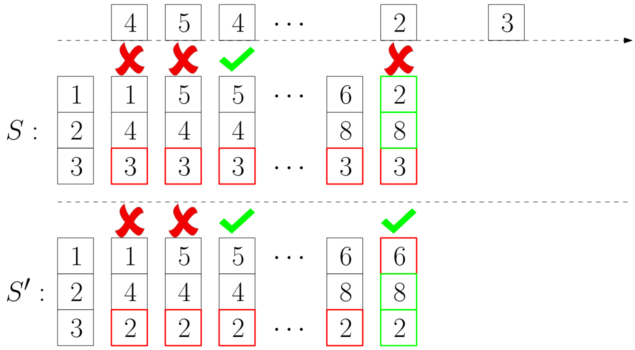

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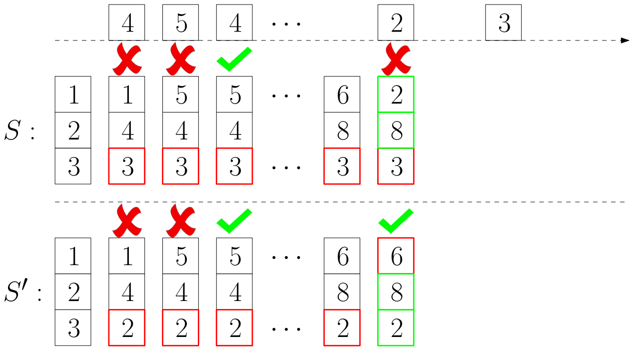

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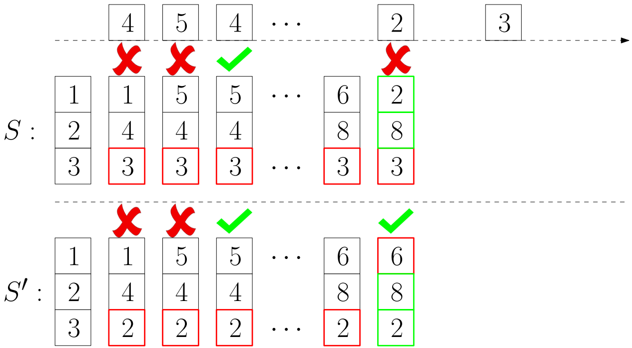
12. We can then guarantee that S' make at most the same number of page-misses as S does.



Proof.

12. We can then guarantee that S' make at most the same number of page-misses as S does.
    - Idea: if S has a page-hit and S' has a page-miss, we use the opportunity to make the status of S' the same as that of S. □

- Thus, we have shown how to create another solution S' with the same number of page-misses as that of the optimum solution S. Thus, we proved

Lemma Assume at time 1 a page fault happens and there are no empty pages in the cache. Let p^* be the page in cache that is not requested until furthest in the future. There is an optimum solution in which p^* is evicted at time 1.

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Lemma Assume at time 1 a page fault happens and there are no empty pages in the cache. Let p^* be the page in cache that is not requested until furthest in the future. It is safe to evict p^* at time 1.

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Lemma Assume at time 1 a page fault happens and there are no empty pages in the cache. Let p^* be the page in cache that is not requested until furthest in the future. It is safe to evict p^* at time 1.

Theorem The furthest-in-future strategy is optimum.

for $t \leftarrow 1$ to T **do**

  if ρ_t is in cache, **then** do nothing

  else if there is an empty page in cache, **then**

    evict the empty page and load ρ_t in cache

  else

    $p^* \leftarrow$ the page in cache that is not used furthest in the future

    evict p^* and load ρ_t in cache

Q: How can we make the algorithm as fast as possible?

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- The running time can be made to be $O(n + T \log k)$.
- For each page p, use a linked list to store the time steps in which p is requested.
  - We can find the next time a page is requested easily.
- Use a priority queue data structure to hold all the pages in cache, so that we can easily find the page that is requested furthest in the future.

for every $p \leftarrow 1$ to n **do**

  $lists[p] \leftarrow$ linked list of times in which p is requested, in increasing order \\\\put ∞ at the end of the list

  $pointer[p] \leftarrow$ head of $lists[p]$

  $nexttime[p] \leftarrow$ value pointed by $pointer[p]$

$Q \leftarrow$ empty priority queue

for every $t \leftarrow 1$ to T **do**

  move $pointer[\rho_t]$ to right by one position

  $nexttime[\rho_t] \leftarrow$ value pointed by $pointer[\rho_t]$

  if $\rho_t \in Q$ **then** Q.update-priority$(\rho_t, nexttime[\rho_t])$, continue

  if Q has size k **then** $p \leftarrow Q$.extract-max() and evict p

  load ρ_t

  add ρ_t to Q with priority value $nexttime[\rho_t]$

Outline

Encoding Letters Using Bits

- 8 letters a, b, c, d, e, f, g, h in a language
- need to encode a message using bits
- idea: use 3 bits per letter

a	b	c	d	e	f	g	h
000	001	010	011	100	101	110	111

$$deacfg \rightarrow 011100000010101110$$

Q: Can we have a better encoding scheme?

- Seems unlikely: must use 3 bits per letter

Q: What if some letters appear more frequently than the others?

Q: If some letters appear more frequently than the others, can we have a better encoding scheme?

A: Using variable-length encoding scheme might be more efficient.

Idea

- using fewer bits for letters that are more frequently used, and more bits for letters that are less frequently used.

Q: What is the issue with the following encoding scheme?

- a: 0 b: 1 c: 00

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Solution

Use prefix codes to guarantee a unique decoding.

Prefix Codes

Def. A prefix code for a set S of letters is a function $\gamma : S \to \{0,1\}^*$ such that for two distinct $x, y \in S$, $\gamma(x)$ is not a prefix of $\gamma(y)$.

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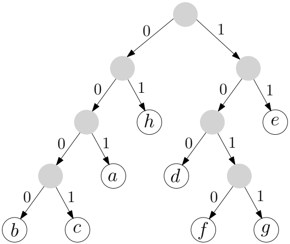
a	b	c	d
001	0000	0001	100
e	f	g	h
11	1010	1011	01



Prefix Codes Guarantee Unique Decoding

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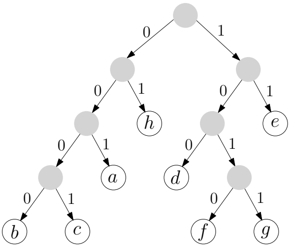

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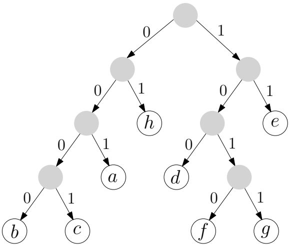

- 0001001100000001011110100001001

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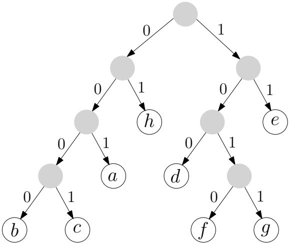

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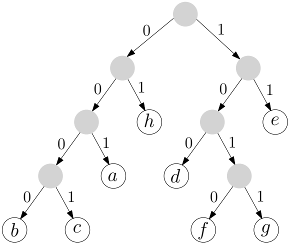

- 0001/001/100000001011110100001001
- ca

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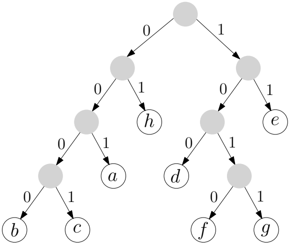
11	1010	1011	01



- 0001/001/100/00000101111010000100
- cad

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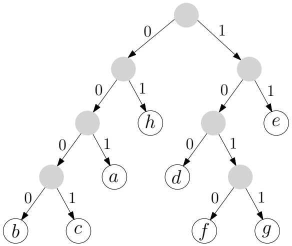

- 0001/001/100/0000/01011110100001001
- cadb

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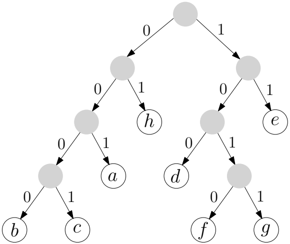

- 0001/001/100/0000/01/01111010000100 1
- cadbh

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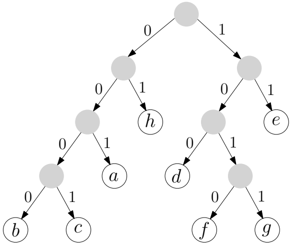

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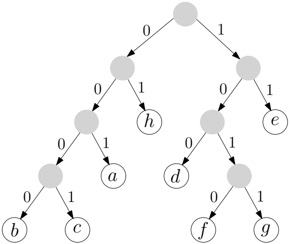

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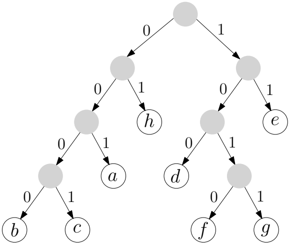

- 0001/001/100/0000/01/01/11/1010/0001001
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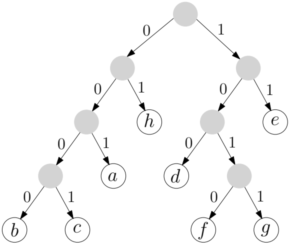

- 0001/001/100/0000/01/01/11/1010/0001/001
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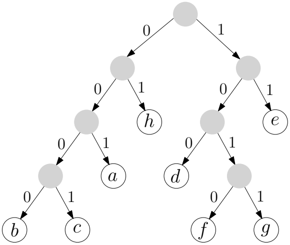
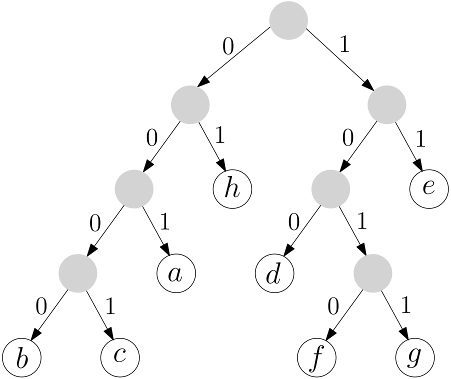
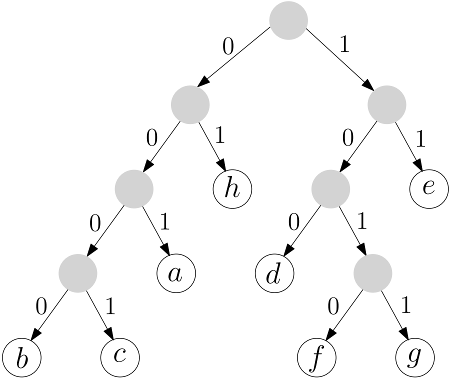


- 0001/001/100/0000/01/01/11/1010/0001/001/
- cadbhhefca

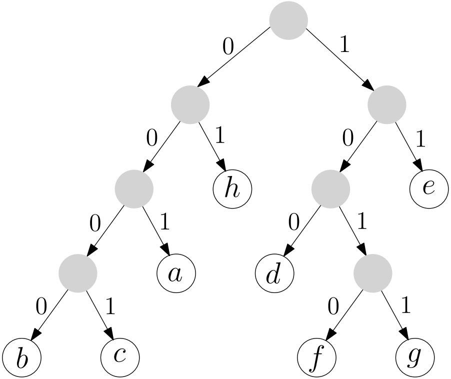


Properties of Encoding Tree



Properties of Encoding Tree

- Rooted binary tree



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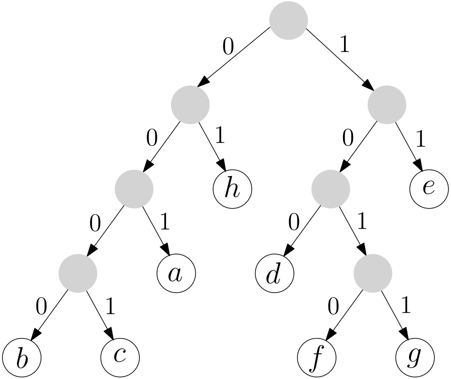

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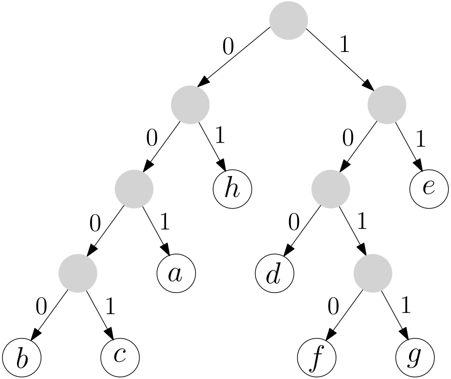

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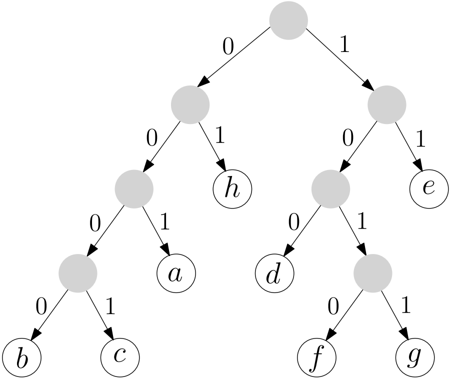

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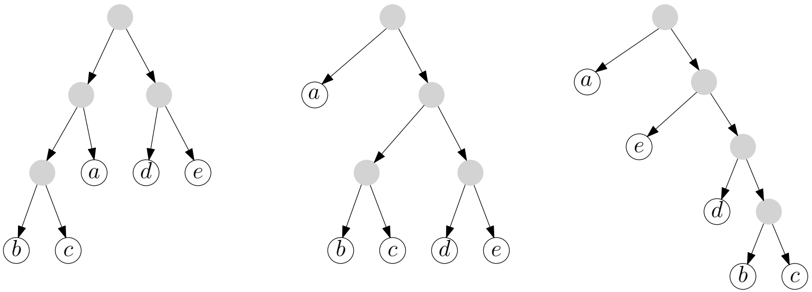
Best Prefix Codes

Input: frequencies of letters in a message

Output: prefix coding scheme with the shortest encoding for the message

example

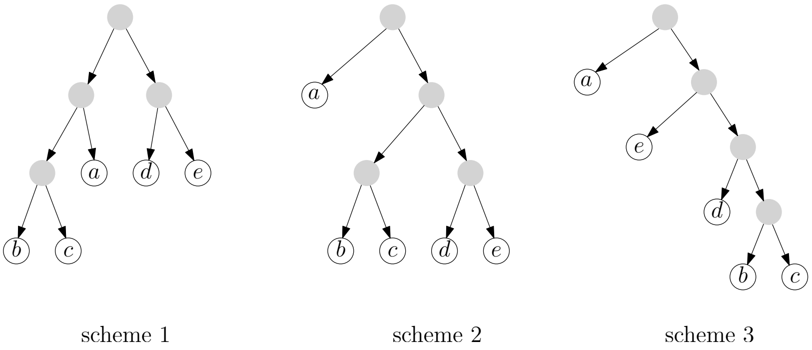
letters	a	b	c	d	e	
frequencies	18	3	4	6	10	



scheme 1 scheme 2 scheme 3

example

letters	a	b	c	d	e	
frequencies	18	3	4	6	10	
scheme 1 length	2	3	3	2	2	total = 89
scheme 2 length	1	3	3	3	3	total = 87
scheme 3 length	1	4	4	3	2	total = 84



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- Hard to design a strategy; residual problem is complicated.
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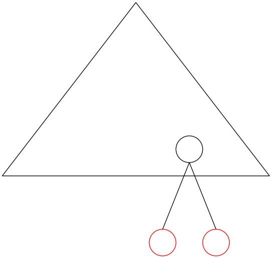
- Not clear how to design the greedy algorithm

A: We can choose two letters and make them brothers in the tree.

Which Two Letters Can Be Safely Put Together As Brothers?

- Focus on the "structure" of the optimum encoding tree



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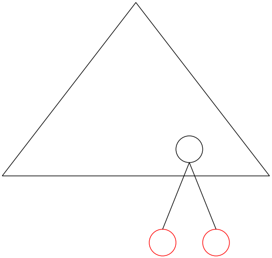
- Focus on the "structure" of the optimum encoding tree
- There are two deepest leaves that are brothers



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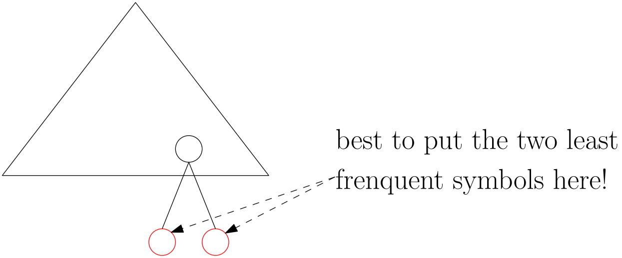

Which Two Letters Can Be Safely Put Together As Brothers?

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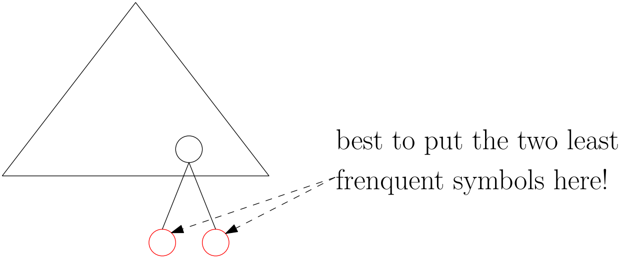

Lemma It is safe to make the two least frequent letters brothers.

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Q: Is the residual problem another instance of the best prefix codes problem?

A: Yes, though it is not immediate to see why.

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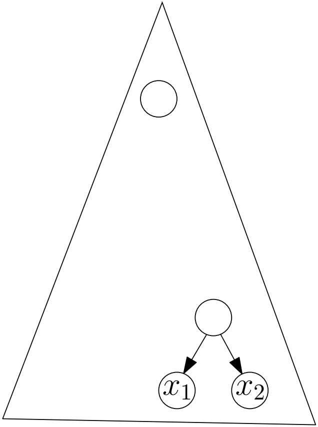
- x_1 and x_2: the two letters we decided to put together.
- d_x the depth of letter x in our output encoding tree.



$$
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& \sum_{x \in S} f_x d_x \\
= & \sum_{x \in S \setminus \{x_1, x_2\}} f_x d_x + f_{x_1} d_{x_1} + f_{x_2} d_{x_2} \\
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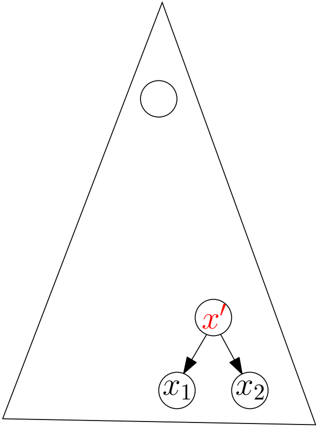

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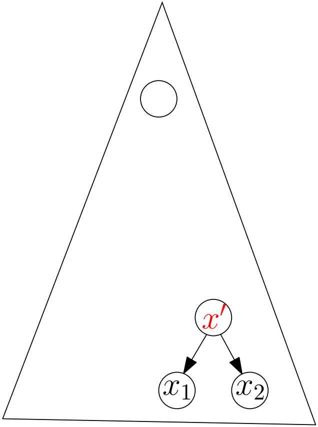

Def: $f_{x'} = f_{x_1} + f_{x_2}$

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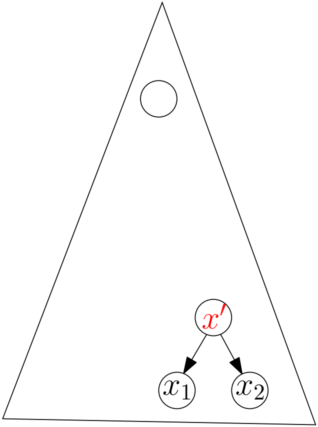

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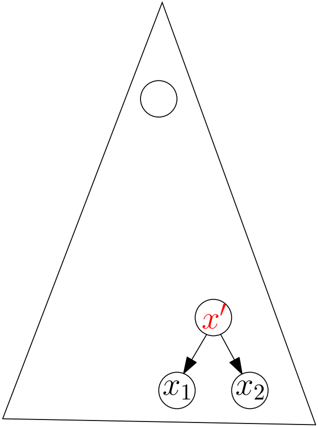

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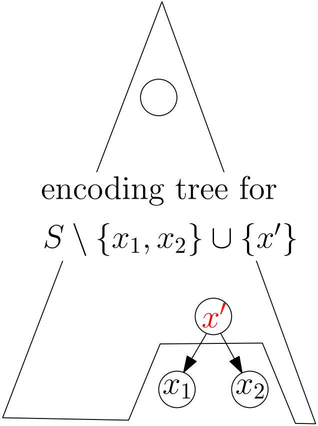

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In order to minimize

$$\sum_{x \in S} f_x d_x,$$

we need to minimize

$$\sum_{x \in S \setminus \{x_1, x_2\} \cup \{x'\}} f_x d_x,$$

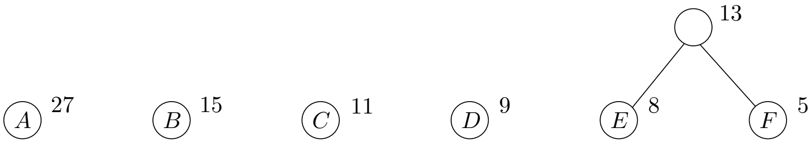
subject to that d is the depth function for an encoding tree of $S \setminus \{x_1, x_2\}$.

- This is exactly the best prefix codes problem, with letters $S \setminus \{x_1, x_2\} \cup \{x'\}$ and frequency vector f!

Example

A 27 B 15 C 11 D 9 E 8 F 5

Example



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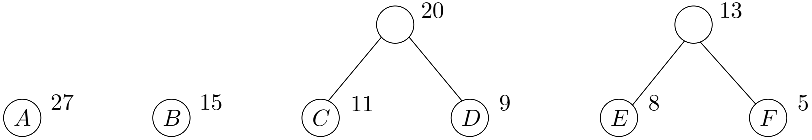

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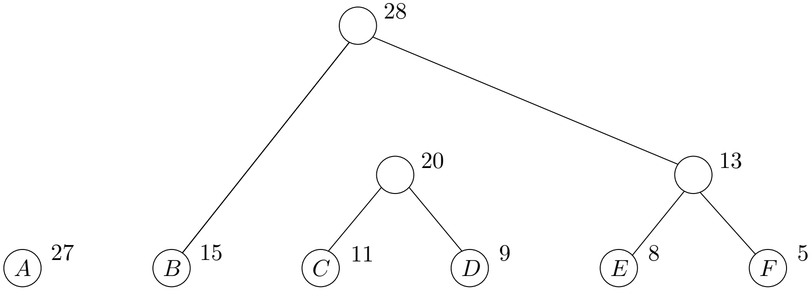

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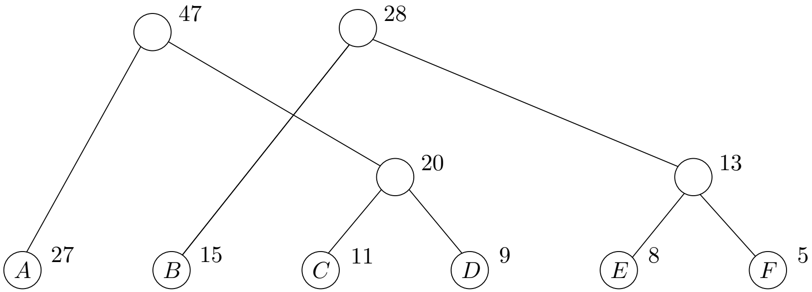

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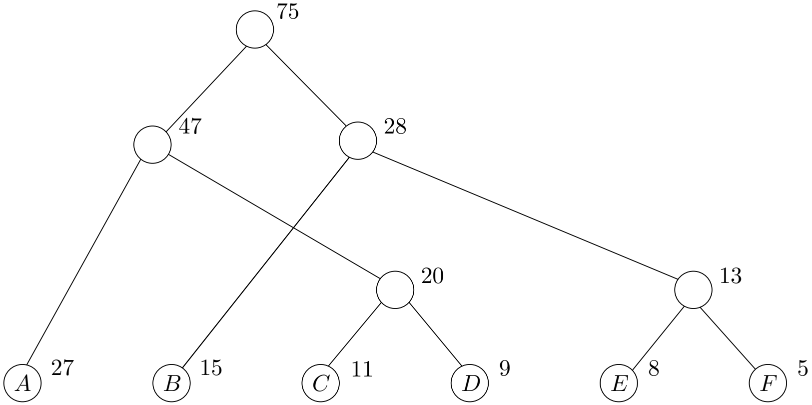

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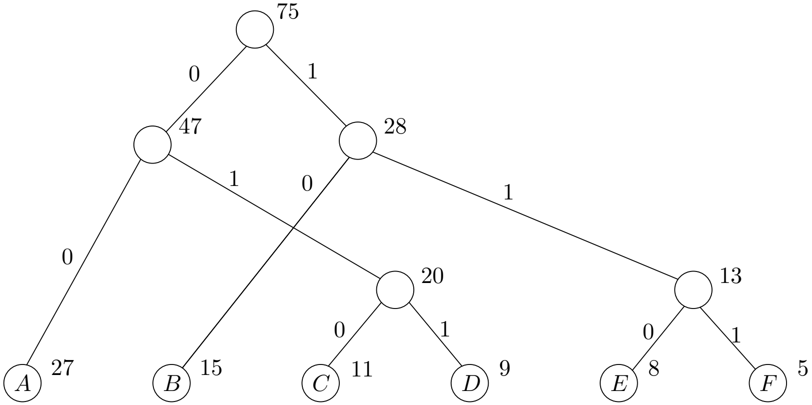

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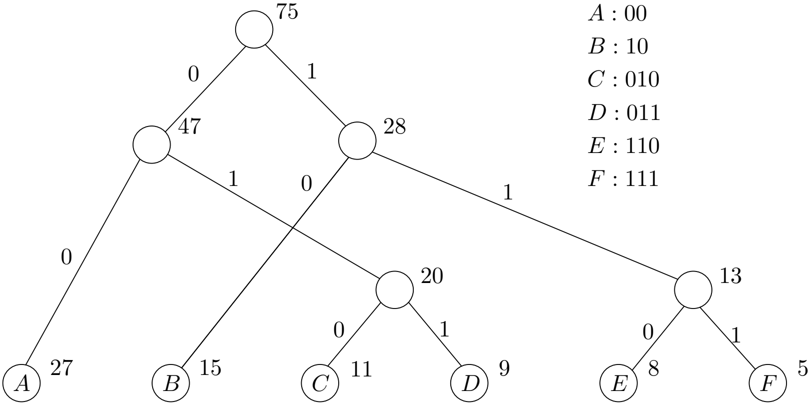

$A : 00$

$B : 10$

$C : 010$

$D : 011$

$E : 110$

$F : 111$

Def. The codes given the greedy algorithm is called the Huffman codes.

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Huffman(S, f)

while $|S| > 1$ **do**

  let x_1, x_2 be the two letters with the smallest f values

  introduce a new letter x' and let $f_{x'} = f_{x_1} + f_{x_2}$

  let x_1 and x_2 be the two children of x'

  $S \leftarrow S \setminus \{x_1, x_2\} \cup \{x'\}$

return the tree constructed

Algorithm using Priority Queue

Huffman(S, f)

```
Q ← build-priority-queue(S)
while Q.size > 1 do
    x_1 ← Q.extract-min()
    x_2 ← Q.extract-min()
    introduce a new letter x' and let f_{x'} = f_{x_1} + f_{x_2}
    let x_1 and x_2 be the two children of x'
    Q.insert(x')
return the tree constructed
```

Outline

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- At each step, make an irrevocable decision using a "reasonable" strategy

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- Offline caching: trivial
- Huffman codes: merge two letters into one