CSE 431/531: Algorithm Analysis and Design (Spring 2020)

NP-Completeness – Recitation

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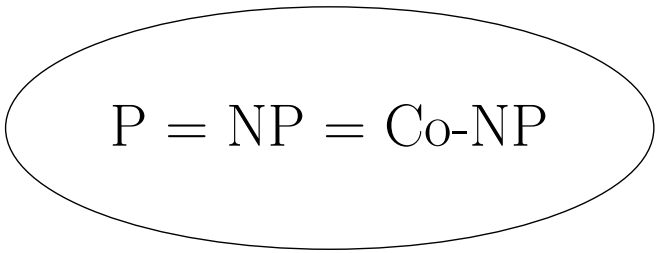
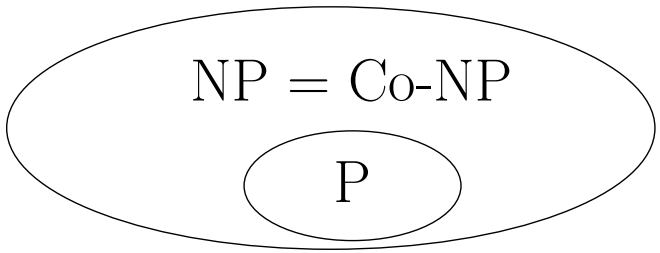
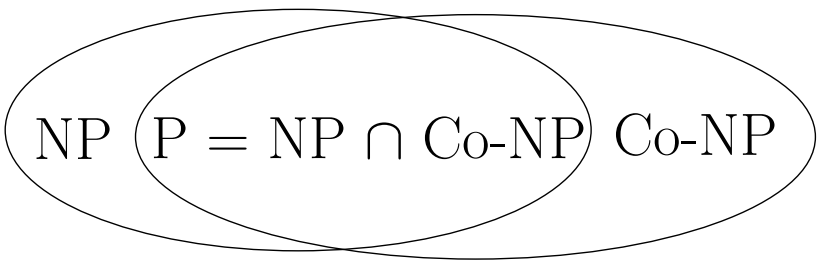
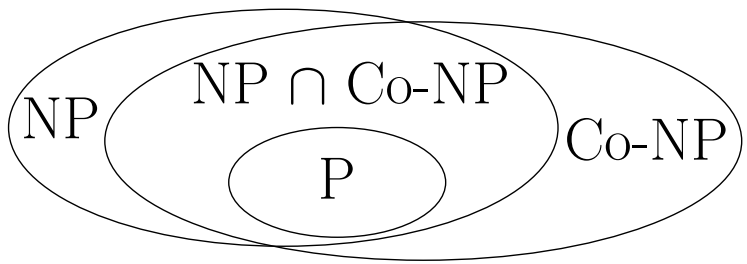
Exercises

Recall the 4 scenarios:









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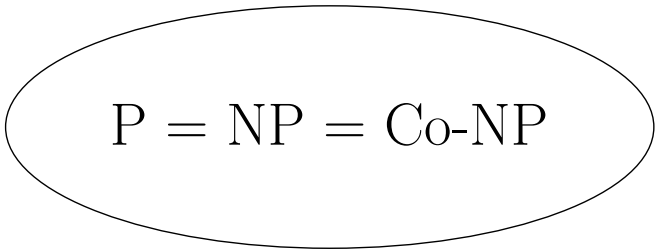

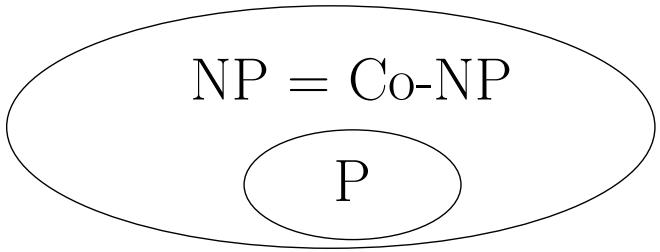

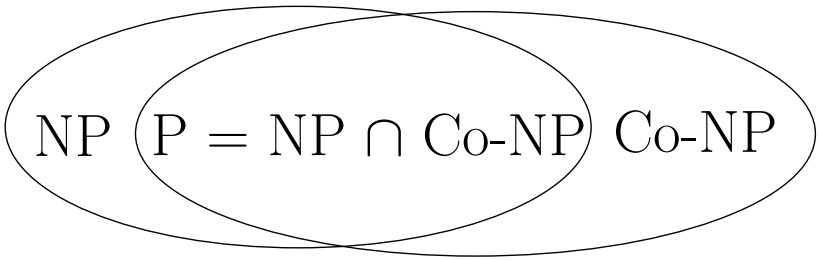

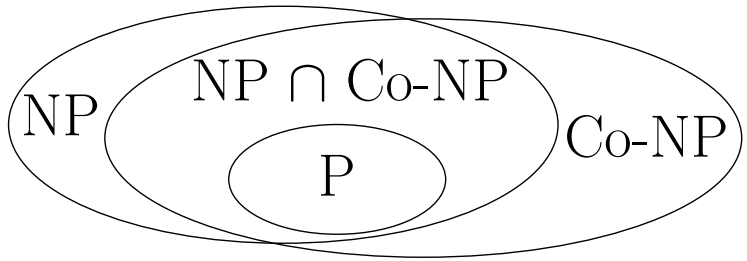

- Prove: P = NP if and only if P = CO-NP

Exercises

For each of the following problem X, answer: whether (1) $X \in$ NP, (2) $X \in$ CO-NP. Each answer is either "yes" or "we do not know".

1. Given a graph $G = (V, E)$, whether G is 4-colorable.

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2. Given a graph $G = (V, E)$ and an integer $t > 0$, whether the minimum vertex cover of G has size at least t.
3. Given a directed graph $G = (V, E)$, with weights $w : E \to \mathbb{R}_{>0}$, $s, t \in V$, and a number $L > 0$, whether the length of the shortest path from s to t in G is at most L.

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4. Given two boolean formulas, whether the they are equivalent. For example, $(x_1 \vee x_2) \wedge (\neg x_1 \vee x_3)$ and $(\neg x_1 \wedge x_2) \vee (x_1 \wedge x_3)$ are equivalent since they give the same value for every assignment of (x_1, x_2, x_3).

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2. Hamiltonian-Cycle $\leq_P$ Hamiltonian-Path
3. Given a graph $G = (V, E)$, the degree-3 spanning tree (D3ST) problem asks whether G contains a spanning tree T of degree at most 3. Prove Hamiltonian-Path $\leq_P$ D3ST.