CSE 431/531: Algorithm Analysis and Design (Spring 2021)

Divide-and-Conquer – Recitation

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Solving Recurrences

For each of the following recurrences, use the master theorem to give the tight asymptotic upper bound.

1. $T(n) = 4T(n/3) + O(n)$. $T(n) = O(\qquad)$
2. $T(n) = 3T(n/3) + O(n)$. $T(n) = O(\qquad)$
3. $T(n) = 4T(n/2) + O(n^2\sqrt{n})$. $T(n) = O(\qquad)$
4. $T(n) = 8T(n/2) + O(n^3)$. $T(n) = O(\qquad)$

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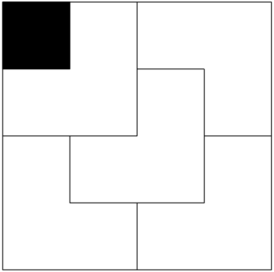
Covering Chessboard using L-shape Tiles

Consider a $2^n \times 2^n$ chessboard with one arbitrary chosen square removed. Prove that any such chessboard can be tiled without gaps by L-shaped pieces, each composed of 3 squares. The following figure shows how to tile a 4×4 chessboard with the square on the left-top corner removed, using 5 L-shaped pieces.



Finding Local Minimum In a 1-D Array

Given an array $A[1 .. n]$ of n **distinct** numbers, we say that some index $i \in \{1, 2, 3 \cdots, n\}$ is a local minimum of A, if $A[i] < A[i-1]$ and $A[i] < A[i+1]$ (we assume that $A[0] = A[n+1] = \infty$). Suppose the array A is already stored in memory. Give an $O(\lg n)$-time algorithm to find a local minimum of A.

Finding Local Minimum In a 2-D Matrix(Hard Problem)

Given a two-dimensional array $A[1 \mathinner{..} n, 1 \mathinner{..} n]$ of n^2 **distinct** numbers, and $i, j \in \{1, 2, \cdots, n\}$, we say that (i, j) is a local minimum of A, if
$A[i,j] < A[i,j-1], A[i,j] < A[i,j+1], A[i,j] < A[i-1,j]$ and $A[i,j] < A[i+1,j]$ (we assume that $A[i,j] = \infty$ if $i \in \{0, n+1\}$ or $j \in \{0, n+1\}$).
Suppose the array A is already stored in memory. Give an $O(n)$-time algorithm to find a local minimum of A.

Integer Multiplication

Given two n-digit integers, output their product. Design a $n^{\log_2 3}$-time algorithm to solve the problem. Notice that you can not multiple two big integers directly using a single operation.

Majority and Weak Majority

Given an array of integers $A[1..n]$, we would like to decide if

1. there exists an integer x which occurs in A more than $n/2$ times. Give an algorithm which runs in time $O(n)$.
2. there exists an integer x which occurs in A more than $n/3$ times. Give an algorithm which runs in time $O(n)$.

You can assume we have the algorithm Select as a black-box, which, given an n-size array A and integer $1 \leq i \leq n$, can return the i-th smallest element in a size n-array in $O(n)$-time.

Median of Two Sorted Arrays

Given two sorted arrays A and B with total size n, you need to design and analyze an $O(\log n)$-time algorithm that outputs the median of the n numbers in A and B. You can assume n is odd and all the numbers are distinct.For example,

- Input: $A = [3, 5, 12, 18, 50]$,
- $B = [2, 7, 11, 30]$,
- Output: 11
- Explanation: the merged set is $[2, 3, 5, 7, 11, 12, 18, 30, 50]$