CSE 431/531: Algorithm Analysis and Design — Spring 2022

Homework 3

Instructor: Shi Li — **Deadline: 4/4/2022**

Your Name: ______________________ Your Student ID: ________________

Problems	1	2	3	4	Total
Max. Score	16	24	25	25	80
Your Score					

The total score for the 4 problems is 90, but your score will be truncated at 80.

Problem 1 For each of the following recurrences, use the master theorem to give the tight asymptotic upper bound. You just need to give the final bound for each recurrence.

(a) $T(n) = 4T(n/3) + O(n)$. $T(n) = O(\_\_\_)$.

(b) $T(n) = 3T(n/3) + O(n^2)$. $T(n) = O(\_\_\_)$.

(c) $T(n) = 4T(n/2) + O(n^2\sqrt{n})$. $T(n) = O(\_\_\_)$.

(d) $T(n) = 8T(n/2) + O(n^3)$. $T(n) = O(\_\_\_)$.

Problem 2 We consider the following problem of counting stronger inversions. Given an array A of n positive integers, a pair $i, j \in \{1, 2, 3, \cdots, n\}$ of indices is called a strong inversion if $i < j$ and $A[i] > 2A[j]$. The goal of the problem is to count the number of strong inversions for a given array A. Give a divide-and-conquer algorithm that runs in $O(n \lg n)$ time to solve the problem.

Problem 3 Given an array A of n **distinct** numbers, we say that some index $i \in \{1, 2, 3 \cdots, n\}$ is a local minimum of A, if $A[i] < A[i-1]$ and $A[i] < A[i+1]$ (we assume that $A[0] = A[n+1] = \infty$). Suppose the array A is already stored in memory. Give an $O(\lg n)$-time algorithm to find a local minimum of A.

Problem 4 Consider a $2^n \times 2^n$ chessboard with one arbitrary chosen square removed. Prove that any such chessboard can be tiled without gaps by L-shaped pieces, each composed of 3 squares. Figure 1 shows how to tile a 4×4 chessboard with the square on the left-top corner removed, using 5 L-shaped pieces. Use divide-and-conquer to solve the problem.

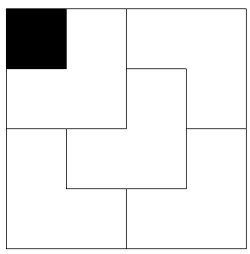

Figure 1: Using 5 tiles to cover a chessboard of size 4×4, with the left-corner missing.