

CSE 431/531: Algorithm Analysis and Design (Fall 2022)
Dynamic Programming – Exercise Problems

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Shortest Path With Even Number of Vertices

- Given a directed acyclic graph with edge weights, our goal is to compute the shortest path from s to t with even number of edges. Reduce the problem to the shortest path problem.

Longest Increasing Subsequence

Given a sequence $A = (a_1, a_2, \dots, a_n)$ of n numbers, we need to find the maximum-length increasing subsequence of A . That is, we want to find a maximum-length sequence (i_1, i_2, \dots, i_t) of integers such that $1 \leq i_1 < i_2 < i_3 < \dots < i_t \leq n$ and $a_{i_1} < a_{i_2} < a_{i_3} < \dots < a_{i_t}$. Design an $O(n^2)$ -time algorithm for the problem.

Counting number of inverted 10-tuples

Given an array A of n numbers, we say that a 10-tuple $(i_1, i_2, \dots, i_{10})$ of integers is inverted if $1 \leq i_1 < i_2 < i_3 < \dots < i_{10} \leq n$ and $A[i_1] > A[i_2] > A[i_3] > \dots > A[i_{10}]$.

- 1 Give an $O(n^2)$ -time algorithm to count the number of inverted 10-tuples w.r.t A .
- 2 Give an $O(n \lg n)$ -time algorithm to count the number of inverted 10-tuples w.r.t A . (Hard Problem.)

Exercise: Counting Number of Domino Coverings

Input: n

Output: number of ways to cover a $n \times 2$ grid using domino tiles

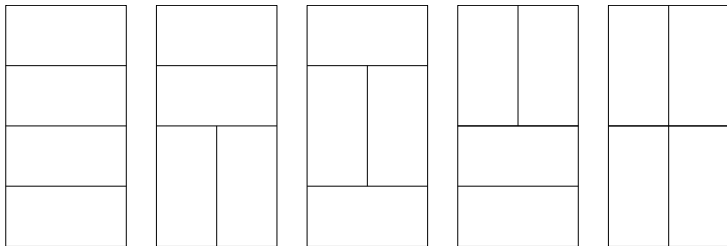


Figure: When n is 4, there are 5 ways to cover the grid.

Maximum weight independent set on trees

Given a tree with node weights, find the independent set of the tree with the maximum total weight.

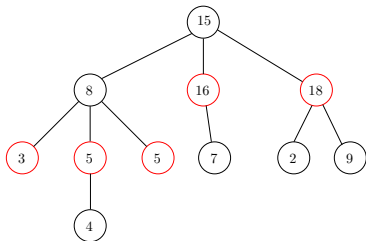


Figure: The maximum-weight independent set of the tree has weight 47. The red vertices give the independent set.

Design an $O(n)$ -time algorithm for the problem, where n is the number of vertices in the tree.