

CSE 431/531: Algorithm Analysis and Design (Fall 2022)

# Dynamic Programming – Exercise Problems

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## Shortest Path With Even Number of Vertices

- Given a directed acyclic graph with edge weights, our goal is to compute the shortest path from  $s$  to  $t$  with even number of edges. Reduce the problem to the shortest path problem.

# Longest Increasing Subsequence

Given a sequence  $A = (a_1, a_2, \dots, a_n)$  of  $n$  numbers, we need to find the maximum-length increasing subsequence of  $A$ . That is, we want to find a maximum-length sequence  $(i_1, i_2, \dots, i_t)$  of integers such that  $1 \leq i_1 < i_2 < i_3 < \dots < i_t \leq n$  and  $a_{i_1} < a_{i_2} < a_{i_3} < \dots < a_{i_t}$ . Design an  $O(n^2)$ -time algorithm for the problem.

## Counting number of inverted 10-tuples

Given an array  $A$  of  $n$  numbers, we say that a 10-tuple  $(i_1, i_2, \dots, i_{10})$  of integers is inverted if  $1 \leq i_1 < i_2 < i_3 < \dots < i_{10} \leq n$  and  $A[i_1] > A[i_2] > A[i_3] > \dots > A[i_{10}]$ .

- ① Give an  $O(n^2)$ -time algorithm to count the number of inverted 10-tuples w.r.t  $A$ .
- ② Give an  $O(n \lg n)$ -time algorithm to count the number of inverted 10-tuples w.r.t  $A$ . (Hard Problem.)

## Exercise: Counting Number of Domino Coverings

**Input:**  $n$

**Output:** number of ways to cover a  $n \times 2$  grid using domino tiles

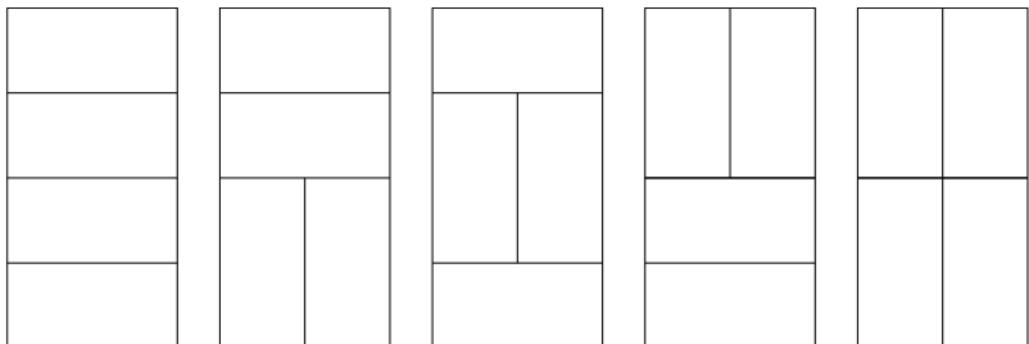
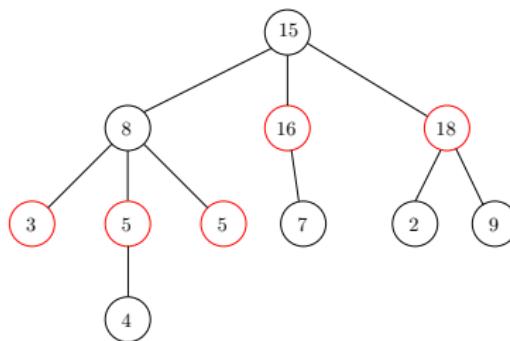


Figure: When  $n$  is 4, there are 5 ways to cover the grid.

# Maximum weight independent set on trees

Given a tree with node weights, find the independent set of the tree with the maximum total weight.



**Figure:** The maximum-weight independent set of the tree has weight 47. The red vertices give the independent set.

Design an  $O(n)$ -time algorithm for the problem, where  $n$  is the number of vertices in the tree.