Given a directed acyclic graph with edge weights, our goal is to compute the shortest path from $s$ to $t$ with even number of edges. Reduce the problem to the shortest path problem.
Longest Increasing Subsequence

Given a sequence $A = (a_1, a_2, \cdots, a_n)$ of $n$ numbers, we need to find the maximum-length increasing subsequence of $A$. That is, we want to find a maximum-length sequence $(i_1, i_2, \cdots, i_t)$ of integers such that $1 \leq i_1 < i_2 < i_3 < \cdots < i_t \leq n$ and $a_{i_1} < a_{i_2} < a_{i_3} < \cdots < a_{i_t}$.

1. Design an $O(n^2)$-time algorithm for the problem.

2. Design an $O(n \log n)$-time algorithm for the problem. (Hard Problem.)
Counting number of inverted 10-tuples

Given an array $A$ of $n$ numbers, we say that a 10-tuple $(i_1, i_2, \cdots, i_{10})$ of integers is inverted if

$1 \leq i_1 < i_2 < i_3 < \cdots < i_{10} \leq n$ and


1. Give an $O(n^2)$-time algorithm to count the number of inverted 10-tuples w.r.t $A$.

2. Give an $O(n \log n)$-time algorithm to count the number of inverted 10-tuples w.r.t $A$. (Hard Problem.)
Exercise: Counting Number of Domino Coverings

Input: \( n \)

Output: number of ways to cover a \( n \times 2 \) grid using domino tiles

Figure: When \( n \) is 4, there are 5 ways to cover the grid.
Maximum weight independent set on trees

Given a tree with node weights, find the independent set of the tree with the maximum total weight.

Figure: The maximum-weight independent set of the tree has weight 47. The red vertices give the independent set.

Design an $O(n)$-time algorithm for the problem, where $n$ is the number of vertices in the tree.