Consider the interval scheduling problem given by a set \( \{1, 2, \cdots, n\} \) of activities, each activity \( i \) with a starting time \( s_i \) and finish time \( f_i \). Decide if the following strategy for designing greedy algorithm is safe of not:

- Select the longest job \( i \) (i.e, the \( i \) with the largest \( f_i - s_i \)). If \( i \) is conflicts with some other job, then we do not schedule \( i \); otherwise we schedule \( i \).
Maximum Independent Set on Trees

Given a tree $T = (V, E)$, find the maximum independent set of the tree. For example, maximum independent set of the tree of following tree has size 7.

![Diagram of a tree with labeled vertices](image)

**Figure:** The red vertices show that the maximum independent set of the tree has size 7.

Design an efficient greedy algorithm to solve the problem.
Weighted Completion Time

Given a set of $n$ jobs $\{1, 2, 3, \cdots, n\}$, each job $j$ with a processing time $t_j > 0$ and a weight $w_j > 0$, we need to schedule the $n$ jobs on a machine in some order. Let $C_j$ be the completion time of $j$ on in the schedule. Then the goal of the problem is to find a schedule to minimize the weighted sum of the completion times, i.e, $\sum_{j=1}^{n} w_j C_j$.

**Example.** Suppose there are two jobs: the first takes time $t_1 = 1$ and has weight $w_1 = 10$, while the second job takes time $t_2 = 3$ and has weight $w_2 = 2$. Then doing job 1 first would yield a weighted completion time of $10 \cdot 1 + 2 \cdot 4 = 18$, while doing the second job first would yield the larger weighted completion time of $10 \cdot 4 + 2 \cdot 3 = 46$. Design an efficient greedy algorithm to solve the problem.
Driving from \( A \) to \( B \) using with minimum number of gas stops

You wish to drive from point \( A \) to point \( B \) along a highway minimizing the time that you are stopped for gas. You are told beforehand the capacity number \( L \) of miles you can drive when the tank is full, the locations \( x_1, \ldots, x_n \) of the gas stations along the highway, where \( x_i \) indicates the distance from the \( i \)-th gas station from \( A \). Design a greedy algorithm to compute the minimum number of times you need to fill the gas tank.
Balanced Strings

A string of “(” and “)” is said to be “balanced”, if it satisfies the recursive definition:

- The empty string “” is balanced.
- If $A$ is balanced then $(A)$ is balanced.
- If $A$ and $B$ are balanced, then $AB$ is balanced.

For example, ”((())())” is balanced.

Problem: Given a string of “(” and “)”, our goal is to remove the minimum number of characters so that the residual string is a balanced.

- Example: ()((())())()
Balanced Strings

A string of “(” and “)” is said to be “balanced”, if it satisfies the recursive definition:

- The empty string “” is balanced.
- If $A$ is balanced then $(A)$ is balanced.
- If $A$ and $B$ are balanced, then $AB$ is balanced.

For example, ”((()))” is balanced.

Problem: Given a string of “(” and “)”, our goal is to remove the minimum number of characters so that the residual string is a balanced.

- Example: $(())((())())$