CSE 431/531: Algorithm Analysis and Design (Spring 2020)
NP-Completeness – Recitation

Lecturer: Shi Li
Department of Computer Science and Engineering
University at Buffalo
Recall the 4 scenarios:

- $P = NP = Co-NP$
- $NP = Co-NP$
- $P = NP \cap Co-NP$
- $NP \cap Co-NP = Co-NP$

Prove: $P = NP$ if and only if $P = Co-NP$
Recall the 4 scenarios:

- \( P = NP = Co-NP \)
- \( NP = Co-NP \)
- \( P = NP \cap Co-NP \)
- \( NP \cap Co-NP \subset Co-NP \)

Prove: \( P = NP \) if and only if \( P = CO-NP \)
Exercises

For each of the following problem $X$, answer: whether (1) $X \in \text{NP}$, (2) $X \in \text{CO-NP}$. Each answer is either “yes” or “we do not know”.

1. Given a graph $G = (V, E)$, whether $G$ is 4-colorable.
Exercises

For each of the following problem $X$, answer: whether (1) $X \in \text{NP}$, (2) $X \in \text{CO-NP}$. Each answer is either “yes” or “we do not know”.

1. Given a graph $G = (V, E)$, whether $G$ is 4-colorable.

2. Given a graph $G = (V, E)$ and an integer $t > 0$, whether the minimum vertex cover of $G$ has size at least $t$. 
Exercises

For each of the following problem $X$, answer: whether (1) $X \in \text{NP}$, (2) $X \in \text{CO-NP}$. Each answer is either “yes” or “we do not know”.

1. Given a graph $G = (V, E)$, whether $G$ is 4-colorable.
2. Given a graph $G = (V, E)$ and an integer $t > 0$, whether the minimum vertex cover of $G$ has size at least $t$.
3. Given a directed graph $G = (V, E)$, with weights $w : E \to \mathbb{R}_{>0}$, $s, t \in V$, and a number $L > 0$, whether the length of the shortest path from $s$ to $t$ in $G$ is at most $L$. 

For example, $(x_1 \lor x_2) \land (\neg x_1 \lor x_3)$ and $(\neg x_1 \land x_2) \lor (x_1 \land x_3)$ are equivalent since they give the same value for every assignment of $(x_1, x_2, x_3)$. 
Exercises

For each of the following problem $X$, answer: whether (1) $X \in \text{NP}$, (2) $X \in \text{CO-NP}$. Each answer is either “yes” or “we do not know”.

1. Given a graph $G = (V, E)$, whether $G$ is 4-colorable.

2. Given a graph $G = (V, E)$ and an integer $t > 0$, whether the minimum vertex cover of $G$ has size at least $t$.

3. Given a directed graph $G = (V, E)$, with weights $w : E \to \mathbb{R}_{>0}$, $s, t \in V$, and a number $L > 0$, whether the length of the shortest path from $s$ to $t$ in $G$ is at most $L$.

4. Given two boolean formulas, whether the they are equivalent. For example, $(x_1 \lor x_2) \land (\neg x_1 \lor x_3)$ and $(\neg x_1 \land x_2) \lor (x_1 \land x_3)$ are equivalent since they give the same value for every assignment of $(x_1, x_2, x_3)$.
Prove the following reductions:

1. **3-Coloring \( \leq_P 4\)-Coloring**
Exercises

Prove the following reductions:

1. 3-Coloring \( \leq_P \) 4-Coloring
2. Hamiltonian-Cycle \( \leq_P \) Hamiltonian-Path
Exercises

Prove the following reductions:

1. 3-Coloring $\leq_P$ 4-Coloring
2. Hamiltonian-Cycle $\leq_P$ Hamiltonian-Path
3. Given a graph $G = (V, E)$, the degree-3 spanning tree (D3ST) problem asks whether $G$ contains a spanning tree $T$ of degree at most 3. Prove Hamiltonian-Path $\leq_P$ D3ST.