Problem 1 (40 points) For each of the following problems, state (1) whether the problem is known to be in NP, and (2) whether the problem is known to be in Co-NP. If your answer is yes, you should briefly describe the efficient certifier.

(a) Given a graph $G = (V, E)$ and $s \leq |V|$, the problem asks whether $G$ contains an independent set of size $s$.

(b) Given two circuits $C_1$ and $C_2$, each with $m$ input variables $z_1, z_2, \cdots, z_m$, decide if the two circuits compute the same function. That is, whether $C_1$ and $C_2$ give the same output for every boolean assignment of $z$-variables.

(c) Given a graph $G = (V, E)$, decide if $G$ is 3-colorable.

(d) Given a graph $G = (V, E)$, decide if $G$ is 2-colorable.

(e) An undirected graph $G = (V, E)$ is called a 1-expander if for every $S \subseteq V$, the number of edges between $S$ and $V \setminus S$ in $G$ is at least $\min\{|S|, |V \setminus S|\}$. Given a graph $G$, decide if $G$ is a 1-expander.

Problem 2 (40 points) In the class, we proved that HP (Hamiltonian Path) $\leq_P$ HC (Hamiltonian Cycle). Prove the other direction, i.e, HC $\leq_P$ HP.